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Analysis of Shock-plugs in Quasi-one-dimensional Compressible Flow

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Analysis of Shock-plugs in Quasi-one-dimensional Compressible Flow

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PASSED X **FAILED** ____

DEDICATION

To the hubris of a young man.

AKNOWLEDGMENTS

I want to thank my friends, family, and the staff at Rose-Hulman. I could not have come this far without all of your love and support.

ABSTRACT

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At small length scales, such as in micro-nozzles, the assumption that a shock wave is infinitesimally thin breaks-down due to the thickness of the shock being non-negligible compared to the dimensions of the nozzle. In this thesis, shock waves of finite thickness, or "shock-plugs," are modeled using the same methods and assumptions as a standard shock wave analysis. Due to the finite thickness of shock-plugs, however, two additional parameters are required in order to account for the differing inlet and exit areas, as well as the pressure on the side walls of the channel. A "typical" micro-nozzle with appropriate dimensions is considered to investigate the effects of these new parameters. It is found that the assumptions made in this model do not constrain shock thickness, and that a shock-plug of any thickness can exist inside of a nozzle for given values of inlet total properties and back pressure. Furthermore, analysis of the pressure on the side walls in the shock-plug model suggests that entropy generation within both shock-plugs and shock waves is best thought of as being due to unrestrained expansion as opposed to internal friction or temperature gradients, as it is more commonly held.

Keywords: shock-plug, shock wave, compressible flow, microflows, microfluidics

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LIST OF ABBREVIATIONS

NASA	National Aeronautics and Space Administration
ESA	European Space Agency

LIST OF SYMBOLS

English Symbols

A	Cross-sectional Area – m^2
a	Speed of Sound – m/s^2
c_p	Constant Pressure Specific Heat – kJ/kg-K
dA	Change in Area – m^2/s
dv	Change in Velocity – m/s^2
h	Enthalpy – kJ/kg
L_s	Length of Shock-plug in x-direction – m
M	Mach Number – Dimensionless
P	Pressure – Pa
P_b	Back Pressure – Pa
P_o	Total Pressure – Pa
P_s	Side Pressure – Pa
R	Specific Gas Constant – Dimensionless
r	Area Ratio – Dimensionless
s	Entropy – kJ/kg-K
s_{gen}	Entropy Generation – kJ/kg-K
T	Temperature – K
T_o	Total Temperature – K
V	Velocity – m/s
v	Velocity – m/s
v	Viscosity – Pa-s

Greek Symbols

γ	Ratio of Specific Heats – Dimensionless
ϱ	Density – kg/m^3
φ	Scaled Side Pressure – Dimensionless

Subscripts

x	Property at the inlet of the shock
y	Property at the exit of the shock
1	Property at the inlet of the shock
2	Property at the exit of the shock

GLOSSARY

Back Pressure – The static pressure at the exit of a converging-diverging nozzle

Continuum Hypothesis – The assertion that a fluid flow can be accurately modeled as a continuous mass, instead of in discrete elements.

Converging-Diverging Nozzle – A gas nozzle designed in such a way that it accelerates gas to above the speed of sound.

Critical Pressure, First – The back pressure at which the flow in the nozzle reaches Mach one at the throat and isentropically decellerates throughout the diverging section of the nozzle.

Critical Pressure, Second – The back pressure at which the flow in the nozzle reaches Mach one at the throat and continues to isentropically accelerate until a standing normal shock wave forms at the exit of the nozzle

Critical Pressure, Third – The back pressure at which the flow in the nozzle reaches Mach one at the throat and isentropically accelerates throughout the diverging section of the nozzle

Isentropic – A process in which the system is adiabatic, frictionless, and reversible

Mean Free Path – An average distance a particle travels between collisions

Micro-nozzle/channel – A nozzle or channel that has dimensions on the scale of µm or nm

Piezoelectric – A system that produces an electric current when under stress or vibration

Rarefaction – A decrease in density of the flow to the point where the continuum hypothesis breaks down

Shock-plug – A rapid change in pressure and speed in a supersonic fluid that occurs over a finite length

Shock Wave – A rapid change in pressure and speed in a supersonic fluid that occurs over an infinitesimally thin length

Total Pressure – The pressure that a flow reaches if it is isentropically slowed to zero velocity

Total Temperature – The temperature that a flow reaches if it is isentropically slowed to zero velocity

Area Ratio – The ratio of inlet area over exit area

1. INTRODUCTION TO STANDING NORMAL SHOCKS

1.1 Introduction to Compressible Flow

One of the classic problems in compressible flow is flow through a nozzle of varying area. The application of the conservation of mass and energy leads to a profound and well known result,

$$\frac{dA}{A} = \frac{dv}{v}\left(1 - M^2\right) \tag{1.1}$$

where A is the area of the nozzle, v is the average velocity of the flow over the change in area, and M is the Mach number of the flow. The Mach number being the ratio between the velocity of the fluid and the speed of sound in that fluid. Equation 1.1 is derived in Moran et. al [1]. The result from Equation 1.1 is that an increase in area can cause the velocity of the flow to either increase or decrease depending on whether the flow is above or below Mach one, the speed of sound of the fluid, respectively.

This result leads to the discussion of converging-diverging nozzles. A converging-diverging nozzle is a nozzle that contains a converging section followed by a diverging section. This nozzle is of particular interest because the Mach number of the flow can reach velocities greater than the speed of sound without external heat tranfer or work, if the flow is at Mach one when the nozzle transitions into the diverging section. This transition point,which has the smallest cross-sectional area, is called the throat of the nozzle. The potential that determines the velocity of the flow is the pressure difference between the inlet and exit of the nozzle. Typically, the inlet pressure of the nozzle is held constant and the back pressure, the pressure at the exit of the nozzle, is varied. Figure 1.1 shows pressure with respect to position in the nozzle as the back pressure is lowered.

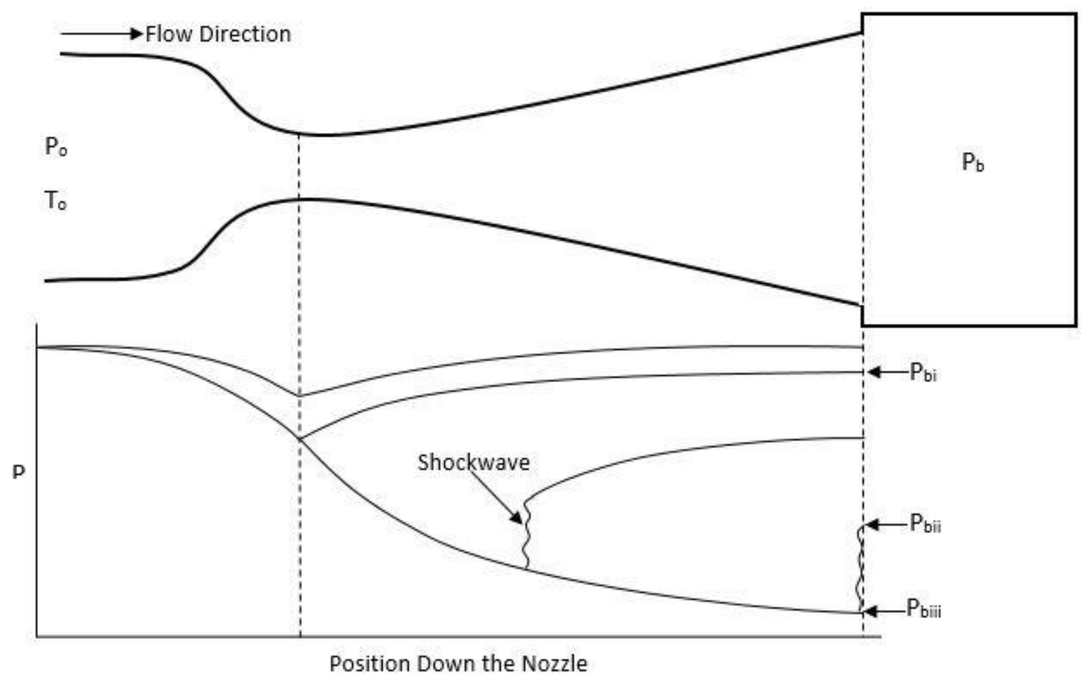

Figure 1.1: Effects of Back Pressure over a Converging-Diverging Nozzle

As the back pressure is decreased slightly, the fluid will speed up until it reaches the throat of the nozzle then it will slow down as the nozzle expands. As the back pressure continues to be lowered, the speed of the flow at the throat increases until it reaches Mach one before it isentropically slows to the exit. The back pressure at which this happens is called the first critical pressure and is shown in Figure 1.1 as P_{bi}. Once the back pressure goes below the first critical pressure the flow at the throat will not increase past Mach one. This phenomenon is called choked flow. Even when the flow is choked, it will accelerate after the throat, due to the fact that an expanding passageway increases the speed of flows above Mach one. There is a back pressure that allows the flow to continue its isentropic acceleration throughout the nozzle. This pressure is called the third critical pressure, P_{biii}. Between the first and third critical pressure, however, the flow cannot isentropically match the back pressure. The flow rapidly decelerates, creating entropy, to meet the back pressure. This rapid deceleration is called a normal shock wave. A normal shock wave slows the flow down rapidly from above Mach one to below Mach one. With the flow below Mach one, it slows as it continues down the passageway until it matches the back pressure. As the back pressure decreases from the first critical pressure, the shock wave will form at the throat and move down the nozzle getting stronger to adjust to the decreasing back pressure. Eventually there is a back pressure that creates a normal shock wave standing at the exit of the nozzle. This back pressure is called the second critcal pressure, P_{bii}. If the back pressure of a nozzle is set to any pressure between the first and second critcal pressure, then a shock wave has to form inside of the nozzle. This region of back pressures is the focus of this thesis.

If the fluid is assumed to be an ideal gas, the relations between the inlet and exit of the shockwave can be derived from first principles. Anderson derives these relations in *Modern Compressible Flow with Historical Perspective*. Anderson assumes that all flow properties are known prior to the shock, the shockwave is adiabatic and infinitesimally thick, and the fluid is an ideal gas with constant specific heats [2]. With these assumptions, Anderson solves for the conditions at the exit of the shockwave by applying conservation of mass, linear momentum, and energy [2].

One of the key results of this derivation is that the flow properties after the shockwave are strictly dependent on the inlet Mach number and the flow after the shock is always subsonic. The most important flow properties are the Mach number, *M*, density of the fluid, ρ, the specfic volume, *v*, the static pressure, *P*, the static temperature, *T*, and the flow enthalpy, *h*. These normal shockwave relations as written by Anderson are:

$$M_2^{\,2} = \frac{1 + \left(\frac{\gamma-1}{2}\right) M_1^{\,2}}{\gamma M_1^{\,2} - \frac{\gamma-1}{2}} \tag{1.2}$$

$$\frac{\rho_2}{\rho_1} = \frac{v_1}{v_2} = \frac{(\gamma+1) M_1^{\,2}}{2 + (\gamma+1) M_2^{\,2}} \tag{1.3}$$

$$\frac{P_2}{P_1} = 1 + \frac{2\gamma}{\gamma+1}\left(M_1^{\,2} - 1\right) \tag{1.4}$$

$$\frac{T_2}{T_1} = \frac{h_2}{h_1} = \left[1 + \frac{2\gamma}{\gamma+1}\left(M_1^{\,2} - 1\right)\right]\left[\frac{2 + (\gamma-1) M_1^{\,2}}{(\gamma+1) M_1^{\,2}}\right] \tag{1.5}$$

The symbol γ is used to represent the ratio of specific heats of the ideal gas. There are some other interesting results from this derivation. The total temperature is constant across the shockwave while the total pressure decreases. These total properties are the properties that the fluid would have if it was isentropically slowed to a velocity of zero. This forms a convienent way to compare two flows at drastically different velocities. The total pressure decreases across the shockwave results from the entropy generated by the shockwave [2].

In truth, a shockwave is not infinitesimally thick. Anderson states that "the shock thickness is usually on the order of a few molecular mean free paths, typically 10^{-5} cm for air at standard conditions" [2]. Granger goes further to derive an equation for the thickness of the shockwave in *Fluid Mechanics.* Granger states that the thickness of the shock wave is the result of the viscosity of the gas [3]. Assuming that the shear stress inside the shockwave balances the inlet and exit pressures, he uses the conservation of linear momentum to derive Equation 1.6.

$$t = \frac{v}{V} \tag{1.6}$$

For this equation, t is the thickness, v is the kinematic viscosity, and V is the velocity of the fluid upstream of the shock wave. As an example, Granger calculates the thickness of a shockwave that is at 100^{o}F, a viscosity of $1.8x10^{-4}$ ft^{2}/s, at a velocity of of 2000 ft/s. This results in a shockwave of $9.0x10^{-8}$ ft, or $3x10^{-6}$ cm [3]. This means, for converging-diverging nozzles with a throat on the order of tens of centimeters, the infinitesimally thin assumption is reasonable. However, as modern technology advances, and with it the ability to create smaller and smaller nozzles, that assumption does not always hold.

1.2 Supersonic Micronozzles

In 2008, Louisos et al. published a paper discussing "Design considerations for supersonic micronozzles" [4]. They said that NASA and the European Space Agency (ESA) are both investigating the use of small satellites, called 'nanosats' for "the next generation of space missions" [4]. These nanosats require even smaller propulsion systems to achieve the planned missions. Louisos states that the thrust these propulsion systems need to have is on the order of one micronewton. These propulsion systems need to be very small to achieve the precise thrust requirements. They state that the area of the throat of one of these micronozzles has to be 9000 μm^{2} squared or $9x10^{-5}$ cm^{2} [4]. If we assume the nozzle to have a square cross sectional area, the throat of the nozzle is approximately $9x10^{-3}$ cm. This length scale is only two orders of magnitude above the mean free path of air at standard conditions. At two orders of magnitude greater than the mean free paths, the assumption that a shockwave standing in the nozzle is infinitesimally thick could begin to break down.

Louisos goes on to discuss the challenges of designing a nozzle at this scale. The main source of difficulty is that the heat tranfer to and from the nozzle and viscous effects of the fluid cannot be considered negligible, as they are in typical shock wave analyses. One other main concern is how well the continuum hypothesis holds. In this case, when the continuum hypothesis is broken, the flow in the micronozzle cannot be accurately described by the standard conservation principles. This is from the effects of individual molecules in the flow which become more and more significant compared to the bulk flow. The parameter for this is the Knudsen number (*Kn*) as defined by $Kn = \Lambda / L$ where Λ is the mean free path and L is a characteristic length. When Knudsen number reaches the order of 0.1, the effects of individual molecules becomes non-negligible; this is called rarefied flow. Louisos states that, for micronozzles, the lower the back pressure the greater the rarefaction of the flow [4]. At higher back pressures, such as the high atmosphere in this case, the effects of rarefaction are negligible [4].

Multiple times, Louisos mentions that this type of flow has not been extensively investigated in literature or in experiments [4]. Resultantly, the most accurate way to predict the performance of a micro-propulsion system is with a numerical model. These numerical models tend to obscure trends and patterns that can bring insight to these systems. In the specific case of the Louisos paper, the micro-nozzles were in the range of back pressures where shockwaves will not form inside of the nozzle. The effects of rarefaction, thermal 'slip' (a discontinutiy in the flow between the wall of the nozzle and the fluid), and viscous forces were of more concern. However, the study of shockwaves inside of the microscale has not been specifically investigated.

1.3 Micro-scale Shock Tube

A shock tube is an instrument used to experimentally simulate the propagation of a shock wave down a passageway. Unlike standing shock waves in nozzles, the shock wave in a shock tube moves down the length of the tube over a series of sensors. Currently, the literature looks at micro-scale shockwaves using shock tubes; investigating them as they propagate down a shock tube microchannel with a constant cross-sectional area. Information on shock waves on this scale are gathered by either numerical computer-aided calculation, such as in Zeitoun et al. [5], or experimentally, such as in Gholamreza and Brouillette[6]. Gholamreza notes that shock waves have many uses on modern microscale applications and goes into detail about manufaturing a long microscale shock tube. The shock tube itself is a narrow, 17 μm wide, channel that is attached to a much larger, 37 mm wide, channel [6]. A shock wave is created in the larger channel by mean of a rupturable membrane separating a high- and low-pressure section. The resulting shock wave travels down the large channel and enters the microscale section. In this microscale section, there is a series of piezoelectric pressure sensors [6]. This method allowed Gholamreza to measure the propagation speed of the shock wave and pressure change in the microchannel [6]. Gholamreza found that the speed and strength of the shock wave decreases as the shock travels down the long passageway. Gholamreza believes this is the result of viscous and thermal effects taking energy away from the shock wave as it travels down the passage way.

The numerical models investigated by Zeitoun are in the same vein as Gholamreza's experiment. Zeitoun compares three numerical solving methods that simulate the propagation of the shock wave through a microchannel [5]. The three solution methods are a Navier-Stokes solver, a Direct Simulation Monte Carlo solver, and a solver that Zeitoun developed using a "relaxation-type kinetic model equation" [5]. All of the simulation methods were used to calculate the shock wave propagation for channels that give a Knudsen number of 0.05 and 0.5

[5]. Zeitoun finds that all three of the solution methods do agree for the tested values, and that the shock wave dissipates in a similar manner to what Gholarmreza had found.

Though these experiments and simulations focus on micro-scale channels, the microchannels that both Gholarmreza and Zeitoun investigate are linear with no area change. By nature of that fact, their models do not investigate the nature of a steady-state shock wave, like that seen in a diverging section of a converging-diverging nozzle. Louisos's analysis on microscale converging-diverging nozzles does not consider a shock forming inside the nozzle because of the conditions in his design. This means that there is a lack of knowledge when it comes to steady-state, or standing, shocks in a channel where the shock wave thickness is non-negligible. This thesis will develop from analytical first principles a model that looks at a converging-diverging micronozzle where the thickness of the shock wave is significant.

2 General Shock-plug Analysis

2.1 Description of Problem

As with all analytical models, there is a list of assumptions made when modeling shocks in compressible flow. The standard shock wave model assumes that the shock wave's thickness is very small compared to the channel. Anderson states that the thickness of a shock wave is approximately on the order of magnitude of the mean free path. This means that the assumption of negligible thickness is reasonable for most cases [2]. Therefore, the shock wave is modeled as a one-dimensional entity with a discontinuity in velocity, pressure, and temperature. Figure 2.1 shows the resulting schematic diagram,

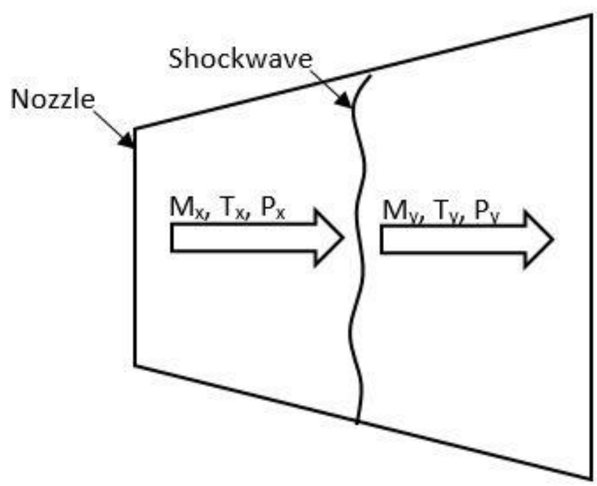

Figure 2.1: Schematic Diagram of the Shock Wave Model

where *M* is the Mach number, *T* is the temperature, and *P* is the pressure of the fluid. Upstream and downstream variables are denoted with *x*- and *y*-subscripts, respectively.

As the length scale of a flow channel grows smaller, however, the thickness of the shock wave becomes non-negligible. In the case of a micro-nozzle, a standing shock would have different cross sectional areas upstream and downstream of the shock, due to the shock wave's thickness. This means that the infinitesimally thin assumption is not valid. Since the inlet and exit area of the shock can no longer be said to be the same, this new shock, called a shock-plug from this point forward, deviates from the classical normal shock relations. The model of a shock-plug adds two new parameters that are critical to investigate. The thickness of the shock-plug, L_s, leads to a change in area between the inlet and exit. This finite thickness also means that there should be a pressure exerted on the system by the side walls of the nozzle, P_s. Since the shock-plug is constrained by the walls of the nozzle, the inlet and exit area of the shock-plug can be defined by a function of length down the nozzle. This allows for a relation between the thickness of the shock-plug, its location in the nozzle, and the change in area of the inlet and exit. Assuming the nozzle's width is defined as a function of the distance from the throat, this relation is straight forward, since the shape of the nozzle is known. The distance from the throat of the nozzle to the leading edge of the shock-plug determines the area of the inlet, A_x. Now, depending on how thick the shock-plug is, the area of the exit, A_y, can be found by adding the thickness of the shock-plug. The schematic diagram of this shock-plug model is shown in Figure 2.2. The same variable naming convention as the shock wave model is used for upstream and downstream conditions.

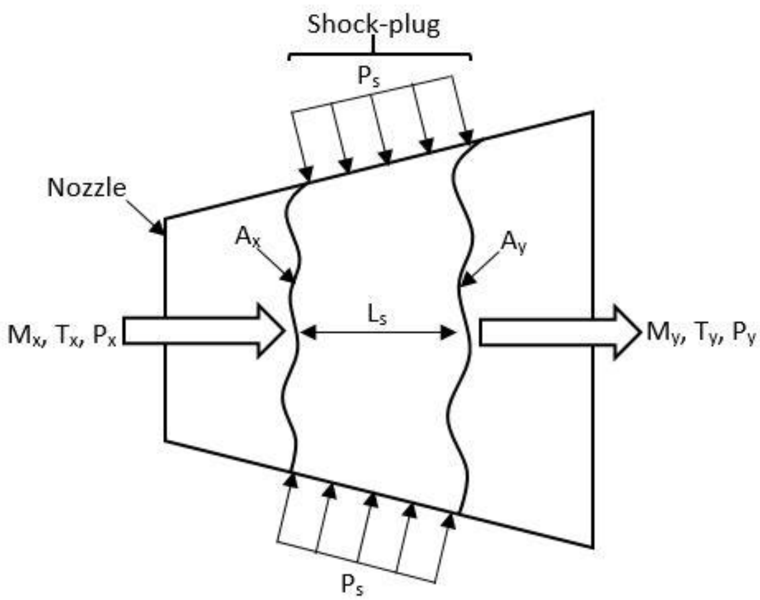

Figure 2.2: Schematic Diagram of the Shock-plug Model

2.2 Shock Wave Derivation

The standing shock wave model is solved by applying continuity, the conversation of linear momentum, and the conservation of energy to the system shown in Figure 2.1. This model also assumes an ideal gas, with constant specific heats, is the working fluid. This leads to the well known shock relations. The most significant revelation from these relations is that all of the downstream flow properties are dependent only on the upstream Mach number, M_x.

Since the geometry of the boundaries of the shock wave are well defined, and the standing shock wave is at steady state, the continuity equation for this model is Equation 2.1.

$$\rho_x A_x v_x = \rho_y A_y v_y \tag{2.1}$$

Here ρ stands for the density of the fluid, A stands for the cross-sectional area, and v is the fluid's velocity. Again, the x- and y- substripts denote the inlet and exit of the shock-plug, respectively. For an ideal gas, the density is $\rho = P/(RT)$, were R is the specific gas constant and T is the temperature of the fluid. The areas are assumed equal in this, shock wave, model. Substituting this into Equation 2.1 gives,

$$\frac{P_x v_x}{RT_x} = \frac{P_y v_y}{RT_y} \tag{2.2}$$

We know that the definition of Mach number is $M = v/a$ and the speed of sound, a, of an ideal gas is $a = \sqrt{\gamma RT}$. Where, γ is the ratio of specific heats as described in Section 1.1. Substituting these two equations into Equation 2.2 leads to

$$P_x M_x \sqrt{\frac{\gamma}{RT_x}} = P_y M_y \sqrt{\frac{\gamma}{RT_y}}\ . \tag{2.3}$$

Rearranging Equation 2.3 yields

$$\frac{M_y}{M_x} = \frac{P_x}{P_y}\sqrt{\frac{T_y}{T_x}}\ . \tag{2.4}$$

The ratios of the pressures and temperatures are found by solving conservation of linear momentum and the conservation of energy equations.

Similar to conservation of mass, the conservation of linear momentum can be described simply. From Figure 2.1, no body forces are present on the shock wave system. This results in the form of the Conservation of Linear Momentum that is shown in Equation 2.5.

$$P_x A_x + \rho_x {v_x}^2 A_x = P_y A_y + \rho_y {v_y}^2 A_y \tag{2.5}$$

Since the shock wave is assumed to be infinitessimally thin, the area terms then cancel. Once again, we use the ideal gas and Mach relations to rearrange Equation 2.5.

This simplifies to,

$$\frac{P_y}{P_x} = \frac{1+\gamma M_x^{\ 2}}{1+\gamma M_y^{\ 2}} . \tag{2.6}$$

The form of the conservation of energy, for this steady-state system is

$$h_x + \frac{v_x^{\ 2}}{2} + q_{in} = h_y + \frac{v_y^{\ 2}}{2} . \tag{2.7}$$

Where h is the specific enthalpy, and q_{in} is the specific heat addition into the system. Assuming there is no heat addition into the system, and the flow is isentropicly decelerated to its total state, Equation 2.7 also states that the total enthalpy h_o is constant, where total enthalpy is defined as, $h_o = h + v^2/2$. The ideal gas assumption means that the enthalpy is strictly a function of temperature, $h_o = f\left(T_o\right)$. Therefore, the resulting total temperature, T_o, of the system is also constant. To get Equation 2.7 into a form that uses temperature instead of enthalpy additional relations are needed. Since this model assumes an idea gas, a change in enthalpy can be expressed as $\Delta h = c_p \Delta T$, where c_p is the specfic heat of the fluid at constant pressure. This relation allows for the change in enthalpy between the inlet and exit to be shown as the change in temperature between the inlet and exit. That last addition piece of information used to put Equation 2.7 in terms of temperature is how c_p, *R*, and γ are related. As mentioned before, γ the ratio of specific heats, specfically, $\gamma = c_p/c_v$. The specfic gas constant, *R*, can be shown to be $R = c_p - c_v$ for an ideal gas. These two relations means that it can be shown that

$c_p = (\gamma R)/(\gamma - 1)$. Using this information as well as the ideal gas and Mach relations used before, conservation of energy can be expressed as:

$$\frac{T_y}{T_x} = \frac{1 + \left(\frac{\gamma - 1}{2}\right) M_x^{\;2}}{1 + \left(\frac{\gamma - 1}{2}\right) M_y^{\;2}} \tag{2.8}$$

Substituting Equtions 2.6 and 2.8 into Equation 2.4 shows that the exit Mach number is a function of solely inlet Mach number. Since the temperature and pressure ratios are functions of inlet and exit Mach number, all of the properties at the exit of the shock wave are dependant upon the inlet Mach number.

2.3 Shock-plug Derivation

The model for the shock-plug is solved by applying continuity, conservation of linear momentum, and conservation of energy to the system shown in Figure 2.2. The same assumptions are used as well. Namely, an ideal gas, with constant specific heats, is the working fluid, and the shock-plug is adiabatic.

Since the geometry of the micro-nozzle and the assumptions of the shock-plug model are the similar, Equation 2.1 can be still be used. Substituting the same ideal gas and Mach relations as before, the continuity equation simplifies into:

$$\frac{M_y}{M_x} = \frac{A_x}{A_y} \frac{P_x}{P_y} \sqrt{\frac{T_y}{T_x}} \tag{2.9}$$

However the areas of the inlet and exit are not assumed to be equal, due to the finite thickness of the shock-plug. This ratio of the areas means that the exit Mach number is not solely dependent on the inlet conditions.

Similarly, we can use Equation 2.5 to apply the conservation of linear momentum to Figure 2.2. The inclusion of side pressure, however, results in an additional term in the linear momentum equation. Note that the nozzle's side wall is assumed to be defined by a smooth function. If we take the pressure on the side wall to be an average value called P_s, the new term for the effect of the side wall pressure in the axial direction is $P_s\left(A_y - A_x\right)$. Adding this term to Equation 2.5 results in,

$$P_x A_x + \rho_x v_x^2 A_x + P_s\left(A_y - A_x\right) = P_y A_y + \rho_y v_y^2 A_y \tag{2.10}$$

After substitutions, similar to what has been done before, and more manipulation, the conservation of linear momentum for a shock-plug becomes,

$$\frac{P_y}{P_x} = \frac{A_x}{A_y}\frac{1+\gamma {M_x}^2}{1+\gamma {M_y}^2} + \frac{P_s}{P_x}\frac{1-\dfrac{A_x}{A_y}}{1+\gamma {M_y}^2} \tag{2.11}$$

Thankfully, there are no aditional terms in the conservation of energy equation. The geometric changes do not affect the form of the equation since both models are assumed to be adiabatic. This means that the same relations, from the shock wave model, will hold for this model. The equation derived for the shock wave model, Equation 2.8, is reproduced below.

(2.8)

2.4 Additional Parameters in Shock-plug Analysis

From the conservation of mass and linear momentum, the change in area between the inlet and exit of the shock-plug is an important new parameter for this model. An area ratio will be defined as,

$$r = \frac{A_y}{A_x} \tag{2.12}$$

Since the shock-plug only reside in the diverging section nozzle,for this model, *r* must be less than or equal to one. As the thickness of the shock-plug increases the *r* parameter will decrease, but it should never truly reach zero, as that would mean the exit area is infinite. The area ratio *r* could be further limited by a case where the resulting shock-plug is unrealizabe.

The pressure on the side wall, the side pressure, can reasonably assumed to be somewhere between the inlet and exit pressures. This is parameterized as

$$P_s = P_x - \varphi\left(P_y - P_x\right), \tag{2.13}$$

where the variable φ is a unitless value that should range from zero to one. If the average value of the side pressure is the same as the inlet pressure, then φ will be zero. If the average value of the side pressure is the same as the exit pressure, then φ will be one. There are no rigid bounds, however. The lower and upper bounds of φ can be determined by two things. Firstly, there is a set minimum to φ imposed by the absolute side pressure. Solving Equation 2.13 for φ and a side pressure of zero leads to $\varphi_{min} = P_x \big/ \left(P_x - P_y\right)$ as the minimum φ for any given inlet and exit pressures. A value of φ below this minimum would create a negative value for the absolute side pressure. This is unrealizable. As with the area ratio, φ could be limited by a case where the resulting shock-plug is unrealizable, such as if the resulting shock-plug destroys entropy.

The parameters *r* and φ can be substituted into Equations 2.9, and 2.11. After some manipulations, these equations become:

$$\frac{M_y}{M_x} = r\frac{P_x}{P_y}\sqrt{\frac{T_y}{T_x}} \tag{2.14}$$

and

$$\frac{P_y}{P_x} = \frac{r(1+\gamma M_x{}^2) + (1-\phi)(1-r)}{(1+\gamma M_y{}^2) - \phi(1-r)}, \tag{2.15}$$

respectively. Equation 2.14 and 2.15 as well as Equation 2.8 form the three primary constraining equations to the shock-plug model.[1]

The application of entropy accounting is also of interest in this model. The discontinuity of the flow properties in the shock wave solution leads to the generation of entropy. Since the shock-plug model is still modeling the same phenomena, although with a finite thickness, the entropy generation is a property that needs to be investigated.

Given the prior assumptions, the entropy accounting for the shock-plug system is,

$$s_{gen} = \Delta s = s_y - s_x \tag{2.16}$$

where s_{gen} is the entropy generated on a per mass basis. Both s_x and s_y terms are the entropy of the fluid, on a per mass basis, of the inlet and exit, respectively. Using ideal gas relations the equation of entropy generation of the shock-plug system is:

$$s_{gen} = c_p \ln\left(\frac{T_y}{T_x}\right) - R\ln\left(\frac{P_y}{P_x}\right) \tag{2.17}$$

If the fluid is reversibly decelerated to a velocity of zero, the fluid has a specific state independent of the velocity. This is known as the total state, with its own total temperature and pressure. This is used to compare two flows that are at different velocities. Using the fact that

[1] A full derivation of the shock-plug model can be found in Appendix A.

Equation 2.8 states the the stagnation temperature is constant, the entropy generation can be shown to be:

$$s_{gen} = -R\ln\left(\frac{P_{o,y}}{P_{o,x}}\right) \tag{2.18}$$

Since the conservation of energy is the same for both shock waves and shock-plugs, it is unsurprising that Equation 2.18 matches the form of the standard shock wave entropy generation.

2.5 Preliminary Results

Equations 2.8 and 2.15 can be substituted into Equation 2.14 to get the exit Mach number in terms of the inlet Mach number, *r*, and *φ*.

$$M_y = \left[\frac{A\gamma(1-B)-1 \pm \sqrt{1-2A\gamma(1-B)+A(\gamma-1)(1-B)^2}}{\gamma-1-A\gamma^2}\right]^{\frac{1}{2}} \tag{2.19}$$

where,

$$A = \frac{M_x^{\,2} r^2 \left[2 + M_x^{\,2}(\gamma-1)\right]}{\left[\left(1+\gamma M_x^{\,2}\right)r + (1-\varphi)(1-r)\right]^2} \tag{2.20}$$

and

$$B = \varphi(1-r)\,. \tag{2.21}$$

The "plus" and "minus" solutions for Equation 2.19 relate to a subsonic and supersonic exit Mach number, respectively. Since this model should converge to the shock-wave results for an area ratio of one, trends that happen in a shock wave, should also happen in a shock-plig. Anderson shows that a normal standing shock wave always has a subsonic solution, therefore a normal standing shock-plug should also use the subsonic solution as well. As a result, only the "plus" solutions will be considered [1].

Using Equation 2.19, the relation between the inlet and exit Mach number can be determined by setting *r* and *φ*. The initial assumption for the pressure change from the inlet and exit is that it is linear. This sets *φ* to be 0.5, by the definition. Figure 2.3 shows the resulting Mach number relations for various area ratios.

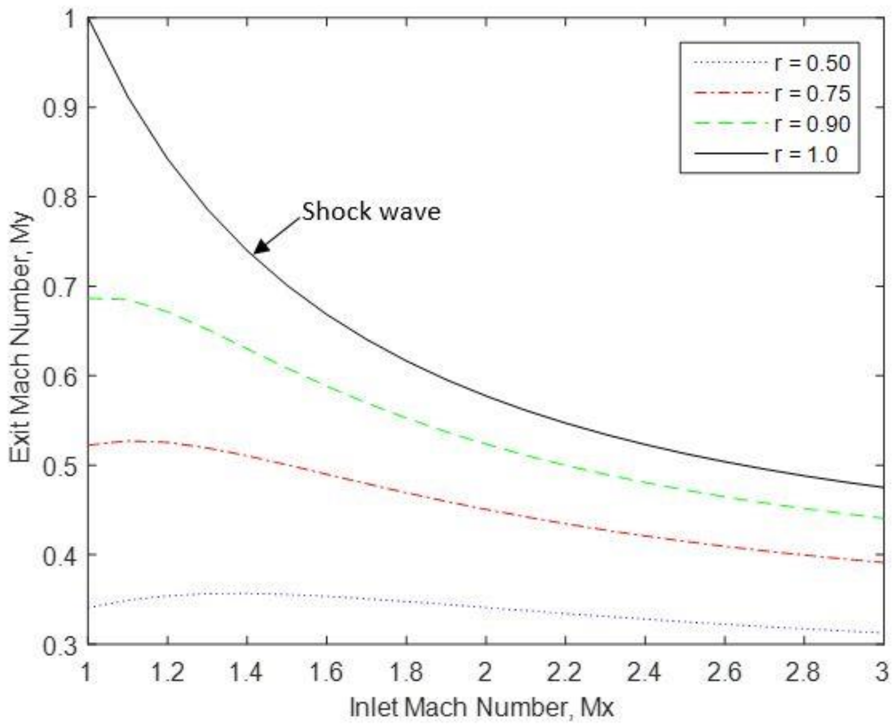

Figure 2.3: Preliminary Relation between Inlet and Exit Mach Number for Various *r* Values

As *r* gets smaller, the area at the exit of the shock-plug gets larger, and the exit Mach number deviates from the shock wave solution corresponding to $r = 1$. At lower supersonic inlet Mach numbers this difference is more pronounced. As the inlet Mach number increases, the effects of *r*, on the exit Mach number is still noticable, but not as prominent.

Figure 2.4 shows the relation between the pressure ratio described in Equation 2.16 and the inlet Mach number. For inlet Mach numbers near one, deviating away from the shock wave solution increases the ratio of exit to inlet pressures. As the inlet Mach number increases, however, smaller area ratios lead to a much smaller pressure increase across the shock.

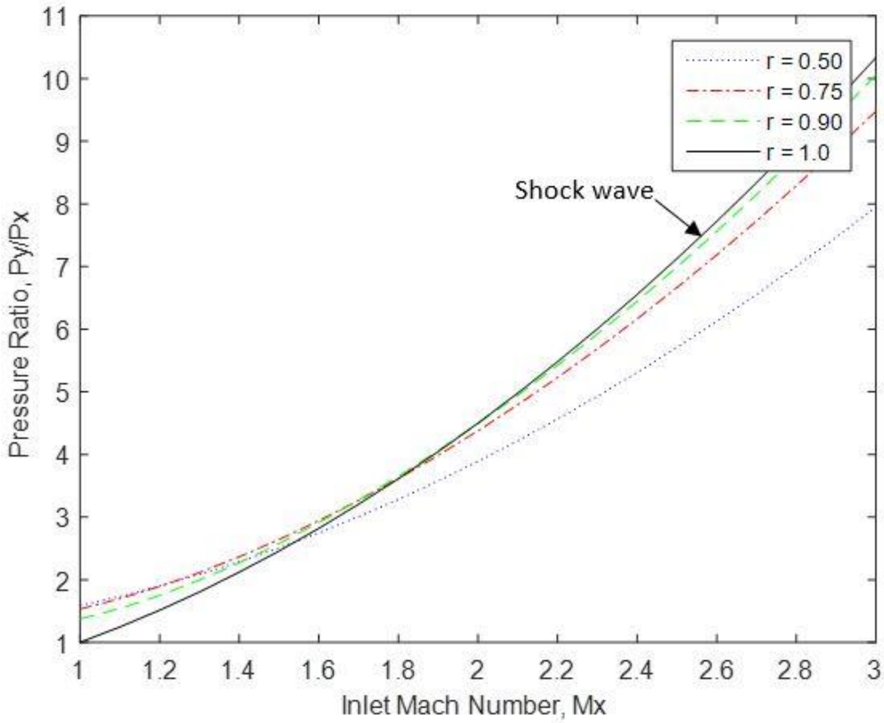

Figure 2.4: Preliminary Relation between Pressure Ratio and Inlet Mach Number for Various *r* Values

Figure 2.5 shows the relation between the temperature ratio and the inlet Mach number. As expected, the change in the temperature ratio is not strongly related to the area ratio. The largest deviations from the shock wave solution in the temperature ratio is at an inlet Mach number of one. The most likely because a shock-plug extends downstream from the and does not become infimtesimally weak like a shock wave does.

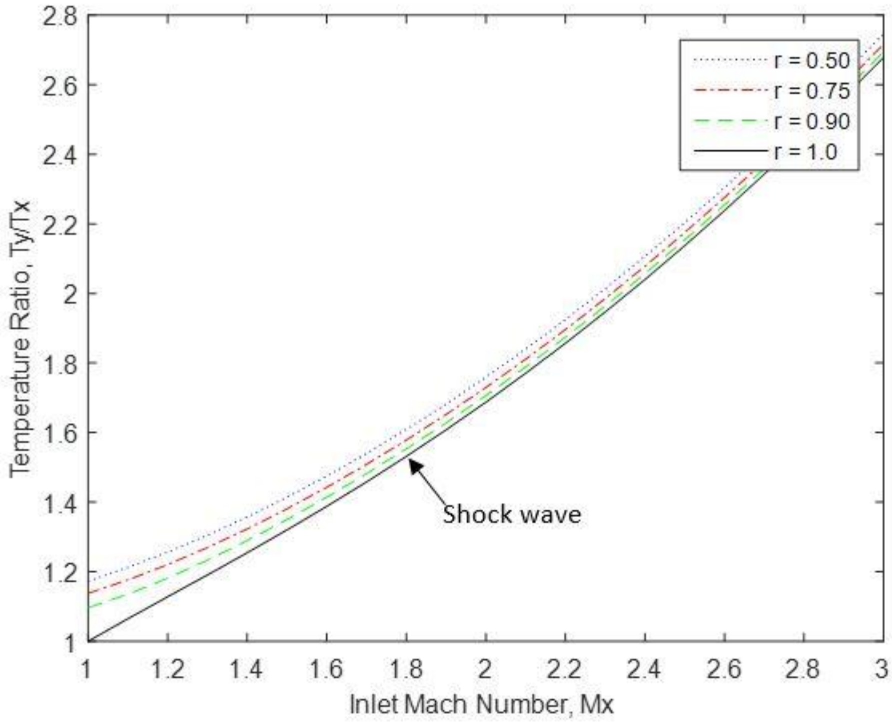

Figure 2.5: Preliminary Relation between Temperature Ratio and Inlet Mach Number for Various *r* Values

Using the information from calculating the pressure and temperature ratios, the entropy generated can be simply found by Equation 2.17. The values for the specific heat and specific gas constant are taken at 20 degrees Celsius [7]. Figure 2.6 shows the resulting entropy generation at various area ratios and inlet Mach numbers.

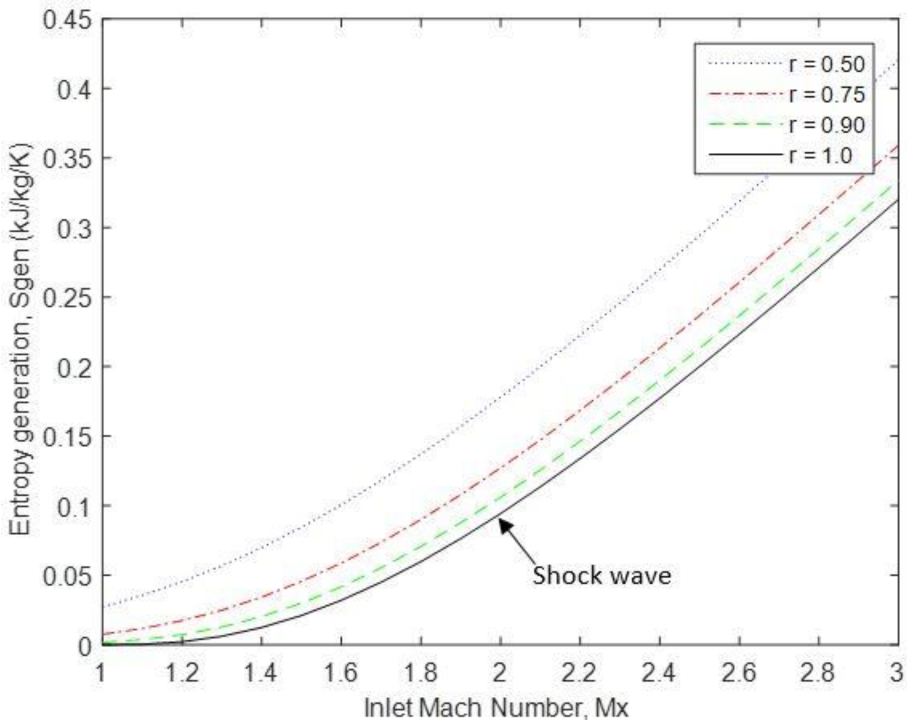

Figure 2.6: Preliminary Relation between Entropy Generation and Inlet Mach Number for Various *r* Values

Since the flow parmeters have a dependence upon φ as well as r, the relationship between φ and the flow parameters needs to be investigated. This was found in a similar way to what was done for the area ratio. For the following graphs, the area ratio will be taken as a constant value, 0.9, and the value of φ is varied over the range -0.5 and 2.0. This area ratio was chosen as it represents a signifigant change between the inlet and exit areas. This gives a wide enough range to investigate where φ corresponds to a zero side wall pressure, and the value of φ for which entropy is destroyed, breaking the second law of thermodynamics. Each figure will also include the predicted values from the standard shock wave realtions.

Figure 2.7 shows the relationship between the inlet and exit Mach numbers for various φ values.

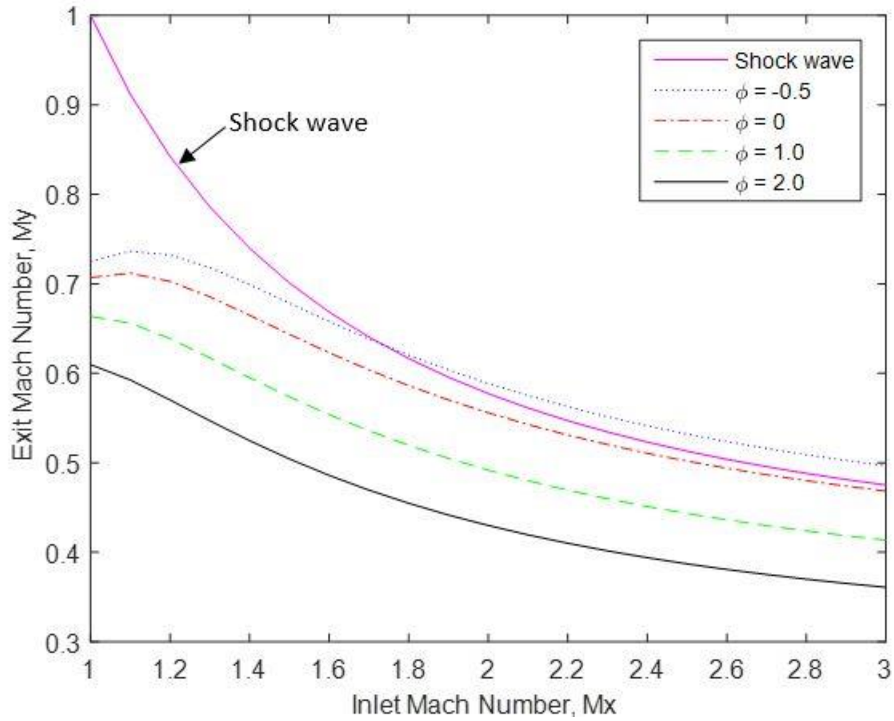

Figure 2.7: Preliminary Relation between Inlet and Exit Mach Number for Various *φ* Values at *r* = 0.9

At low supersonic Mach numbers the shock-plug cases investigated underpredict the exit Mach number. As φ is made more negative the exit Mach number increases towards the shock wave solution. As the inlet Mach number increases, however, the shock wave solution approaches the φ equals zero case. As the inlet Mach number continues to increase, the exit Mach number seems to become a function of predominately φ. It should be noted that the intersection between the $\varphi = -0.5$ case and the shock wave solution is around Mach 1.73. This means that at that inlet Mach number, an area ratio, *r*, of 0.9 and a φ of -0.5 will produce the same exit Mach number as a shock wave at the same inlet conditions.

The relation between inlet Mach number and the pressure ratio is shown in Figure 2.8. As the φ value decreases, the pressure ratio decreases for all inlet Mach numbers. At low inlet speeds all of the investigated cases lead to greater pressure ratios than the shock wave solution. As the inlet speed increases, the shock wave solution crosses the shock-plug cases of $\varphi = -0.5$ and $\varphi = 0.0$ at Mach numbers 1.3 and 1.43, respectively.

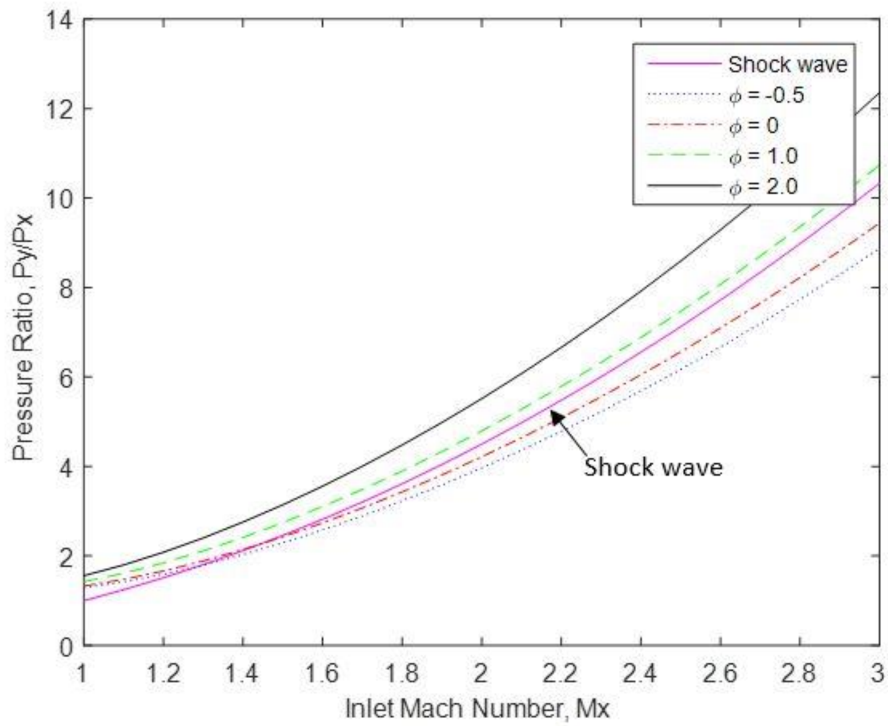

Figure 2.8: Preliminary Relation between Pressure Ratio and Inlet Mach Number for Various φ Values at r = 0.9

The relation between the inlet Mach number and the temperature ratio is shown in Figure 2.9. At small inlet Mach numbers, all of the cases for φ lead to a larger temperature ratio than the shock wave relations. Similarly to Figure 2.7, the shock wave solution converges close to the *φ=0* case as the inlet Mach number increases, and intersects the case where *φ=-0.5* at an inlet Mach number of approximately 1.73.

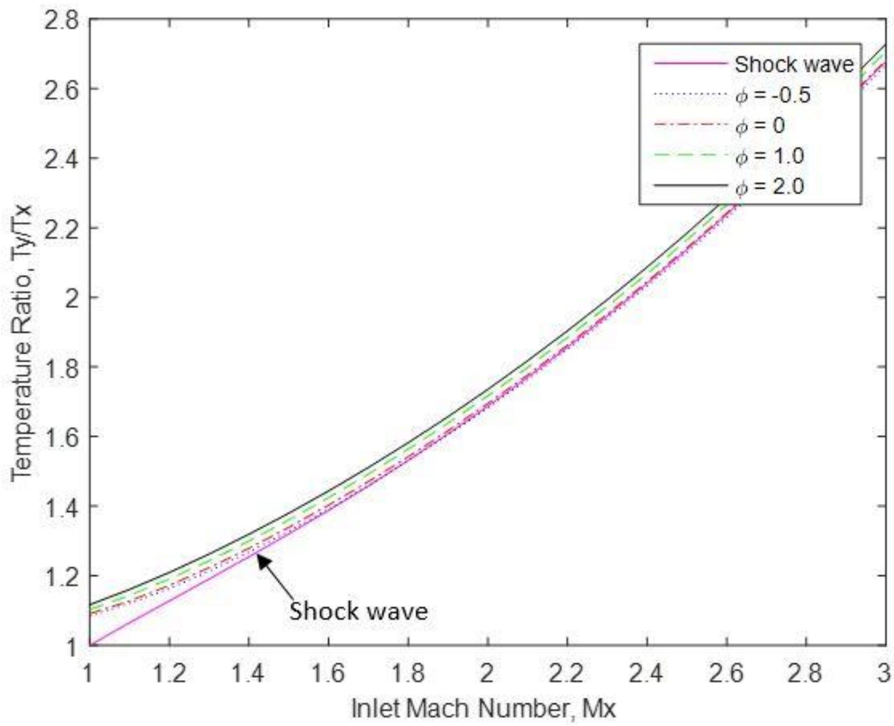

Figure 2.9: Preliminary Relation between Temperature Ratio and Inlet Mach Number for Various *φ* Values at *r* = 0.9

The entropy generated by the shock-plug at various values for the inlet Mach number and *φ* is shown below in Figure 2.10. Both the $\varphi = 2.0$ and $\varphi = 1.0$ cases are unrealizable at low Mach numbers because they destroy entropy at this given area ratio of 0.9. This means that there is a maximum value for *φ* at any given inlet Mach number and area ratio. As *φ* becomes negative, the entropy generation approaches the shock wave value, though never creates the same entropy over the investigated range. At this time, it is hard to determine what value of *φ* will produce the shock wave solution's entropy generation. Further decreasing the φ value is unrealizable due to the fact that it generates a negative side wall pressure as discussed later.

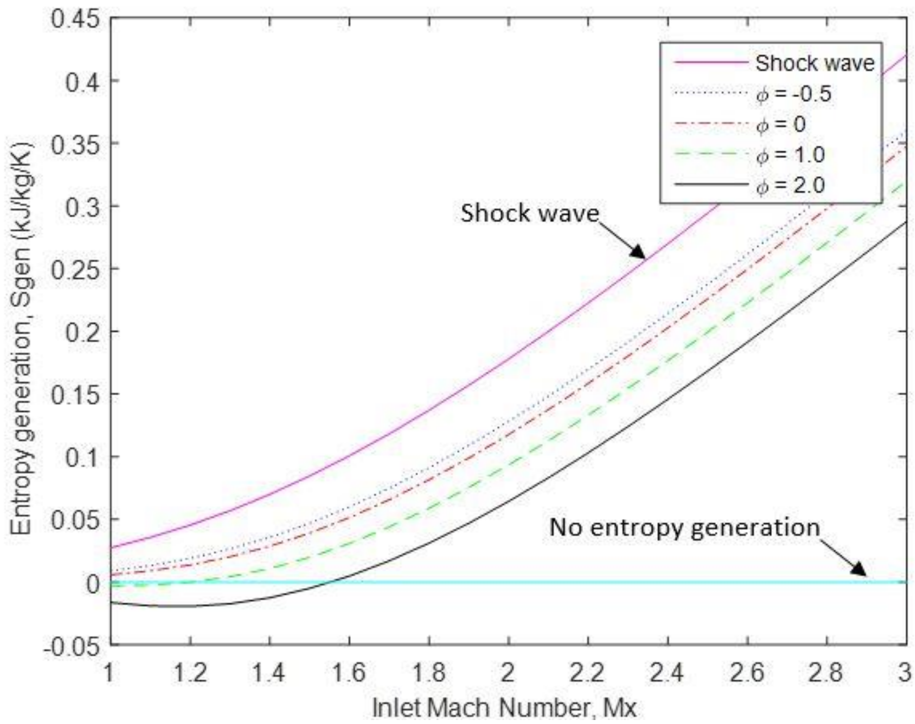

Figure 2.10: Preliminary Relation between Entropy Generation and Inlet Mach Number for Various φ Values at r = 0.9

There is an additional parameter that is specific to φ, side pressure. The ratio of the side pressure to inlet pressure created by the various inlet Mach numbers and φ values is shown below in Figure 2.11.

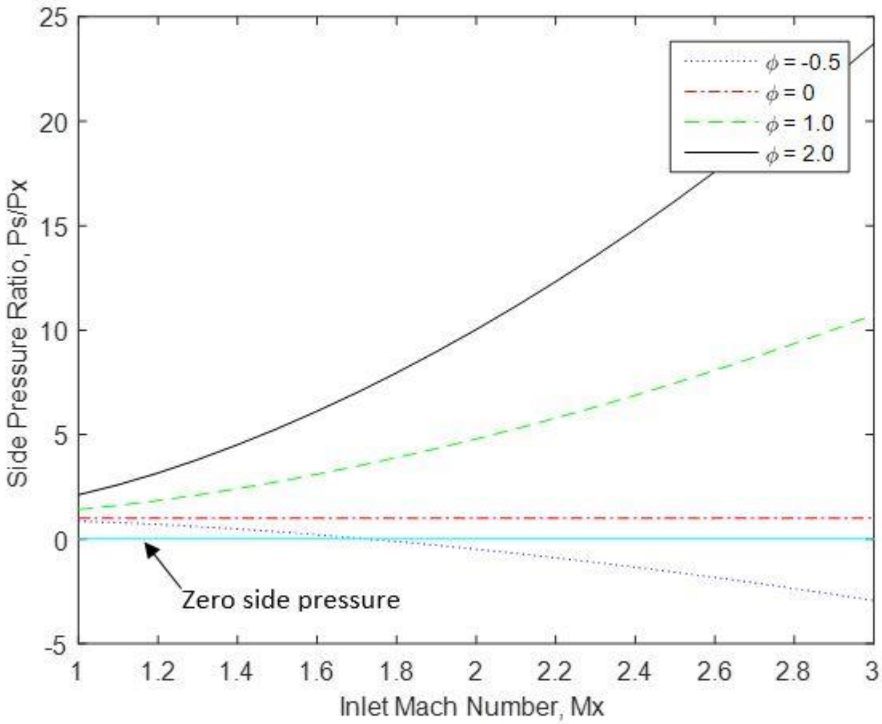

Figure 2.11: Preliminary Relation Between the Side Pressure and Inlet Mach Number for Various *φ* Values at *r* = 0.9

If φ is zero, the side pressure is simply the inlet pressure, P_x a known constant. If φ is negative, the side pressure decreases as the inlet Mach number increases. Eventually the side pressure reaches absolute zero. This means that the interior of the shock-plug is a vacuum and any lower values are not possible. There is, therefore, a minimum value that φ can obtain for any given inlet Mach number and area ratio. With an inlet Mach number of about 1.73 and an area ratio of 0.9, the minimum value that φ can obtain is -0.5. This case is also where the exit Mach number and Temperature ratios matched the shock wave solution. This means that the shock wave and shock plug solution partially match when the absolute side wall pressure zero.[2]

[2] The MATLAB code and data used to create Figures 2.3 through 2.11 can be found in Appendix B.

From Figures 2.7 to 2.11, it can be seen that the parameter φ plays a signifigant role in determining the exit flow parameters of the shock-plug. Since the current literature does not look at the side wall pressure or φ variables when analyzing flows in a channel or with shock waves, a specfic look at this new parameter is needed.

3. Effect of Sidewall Pressure of Shock-plugs

3.1 System

In the classical approach to analyzing quasi-one-dimensional flow within a variable area channel, the properties in the direction of the flow are determined with the conservation of mass and energy as well as entropy accounting equations. When a shock wave is present in the flow, however, the conservation of linear momentum equation is needed to fully solve the flow parameters. Current literature does not investigate linear momentum in an isentopic compressible flow since the system can be fully constrained by the conservation of mass, conservation of energy and entropy accounting . Due to the importance of the side wall pressure in this shock-plug model, further investigation is needed. Information about the side pressure characteristics in flows without any shocks can provide insight into what the side pressure will be in flows with shocks-plugs.

It is important to note that there are some assumptions made when using the variable φ to describe the side wall pressure in a flow. Recall that φ was defined in Equation 2.13;

$$P_s = P_x + \varphi\left(P_y - P_x\right) \tag{2.13}$$

The most important limitation of φ is that it is an average over the test section. φ is also scaled in such a way that it is zero when the side wall pressure is equal to the inlet pressure of the system boundary and one when the side wall pressure is equal to the exit pressure of the system boundary. Values of φ outside of zero or one are unexpected, although not mathmatically impossible.

Before any equations are used to solve for the side wall pressure or φ, a system must first be defined. For the purpose of this analysis, the system must be a smooth nozzle that increases in area as the flow moves downstream. Figure 3.1 shows this system. The flow into and out of the system is labeled as x and y respectively to keep the same notation as the shock-plug model. The values of the inlet Mach number, M_x, ratio of specific heats, γ, total pressures, $P_{o,x}$ and $P_{o,y}$, and both inlet and exit area, A_x and A_y, are assumed to be known.

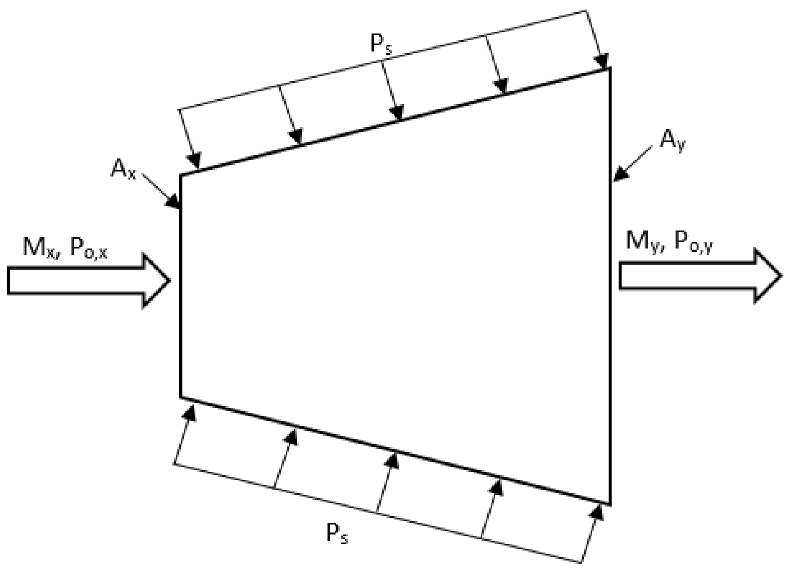

Figure 3.1: Schematic Diagram for the Analysis of the Side Wall Pressure

3.2 Math Model

The conservation of mass, energy, and linear momentum are all needed to solve for the the side wall pressure and then φ. The conservation of mass used, in conjunction with the conservation of energy, the ideal gas law, and the total property relations in Anderson [2], to determine the exit Mach number of the system. The total property relations that are used to make these equations are:

$$\frac{P_o}{P} = \left[1 + \left(\frac{\gamma - 1}{2}\right) M^2\right]^{\gamma/\gamma - 1} \tag{3.1}$$

and

$$\frac{T_o}{T} = 1 + \left(\frac{\gamma - 1}{2}\right) M^2 \tag{3.2}$$

Substituting Equations 3.1 and 3.2 into Equation 2.3 and combining like terms leads to the version of conservation of mass used for this problem, and it is shown below in Equation 3.3.

$$\sqrt{\frac{\gamma}{RT_{o,x}}} \frac{P_{o,x} A_x M_x}{\left(1 + \frac{\gamma - 1}{2} M_x^2\right)^{-(\gamma+1)/[2(\gamma-1)]}} = \sqrt{\frac{\gamma}{RT_{o,y}}} \frac{P_{o,y} A_y M_y}{\left(1 + \frac{\gamma - 1}{2} M_y^2\right)^{-(\gamma+1)/[2(\gamma-1)]}} \tag{3.3}$$

Several simplifications can be made from this form of conservation of mass. The largest simplification is that the total temperature of the system does not change, since there is no heat addition. This means that the $\sqrt{\gamma / RT_{o,x}}$ and the $\sqrt{\gamma / RT_{o,y}}$ terms cancel each other out. As was done prior, the area of the outlet of this system is divided through and the variable *r* is defined as $r = A_x / A_y$. The last simplifying variable definition is the total pressure ratio, P_r . P_r is defined as $P_r = P_{o,y} / P_{o,x}$, so that any drop in the total pressure can be shown as a number from 0 to 1. This means that if the total pressure is reduced by 10 percent, P_r would be approximately 0.9. Applying these simplifications and rearranging Equation 3.3 leads to Equation 3.4.

$$\frac{r}{P_r} \frac{M_x}{\left[1 + \left(\frac{\gamma - 1}{2}\right) M_x^2\right]^{-(\gamma+1)/[2(\gamma-1)]}} = \frac{M_y}{\left[1 + \left(\frac{\gamma - 1}{2}\right) M_y^2\right]^{-(\gamma+1)/[2(\gamma-1)]}} \tag{3.4}$$

Equation 3.4 is the equation that is used to calculate the exit Mach number of the system described in Figure 3.1.

Once the exit Mach number is known, The Mach relations can be used to calculate the static pressure of both the inlet and the exit of the nozzle section. With the static pressure, Mach number, and area of both the inlet and the exit known, the conservation of linear momentum from the shock-plug derivation can be used to find the side wall pressure. Equation 3.1 is substituted into Equation 2.10 so that the system is put into known quantities. Equation 2.10 is reproduced below.

$$P_x A_x + \rho_x v_x^2 A_x + P_s \left(A_y - A_x \right) = P_y A_y + \rho_y v_y^2 A_y . \tag{2.10}$$

Equation 3.5 is the resulting form of the conservation of linear momentum that is used to calculate the side wall pressure, P_s.[3]

$$P_x A_x + \frac{\gamma P_{o,x} A_x M_x{}^2}{\left(1+\frac{\gamma-1}{2}M_x{}^2\right)^{\gamma/(\gamma-1)}} + P_s(A_y - A_x) = P_y A_y + \frac{\gamma P_{o,y} A_y M_y{}^2}{\left(1+\frac{\gamma-1}{2}M_y{}^2\right)^{\gamma/(\gamma-1)}} \tag{3.5}$$

Recall that $P_s = P_x + \varphi\left(P_y - P_x\right)$, the parameter φ is simply found after the side wall pressure is known.

Results of Sidewall Pressure

The first case for investigating the side wall pressure is the isentropic case. Since the flow is isentropic throughout the nozzle segment, the total pressure ratio, P_r, is one. This means that the side wall pressure is determined by the area ratio, *r*, and the inlet Mach number, M_x. Solving Equation 3.3 for the exit Mach number provides two real solutions, because of the $M_y{}^2$ terms. Since the flow must be real, the imaginary solutions are not considered. This leaves the two real

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[3] This algorithm was implemented in the code shown in Appendix C

solutions. One is greater than the other, and the other is less than one. The system for this problem is a nozzle passageway no throat, the flow can never cross Mach one inside of the passageway. This means that the exit Mach number, solved for in Equation 3.4, will match the flow regime of the inlet. If the inlet flow is subsonic, the exit should be subsonic as well. If the inlet flow is supersonic the exit flow should be supersonic as well. By varying the parameters *r* and M_x, the differences in the side wall pressure, and the corresponding *φ* can be seen. Figure 3.2 shows the *φ* value for systems with different inlet Mach numbers and area ratios[4].

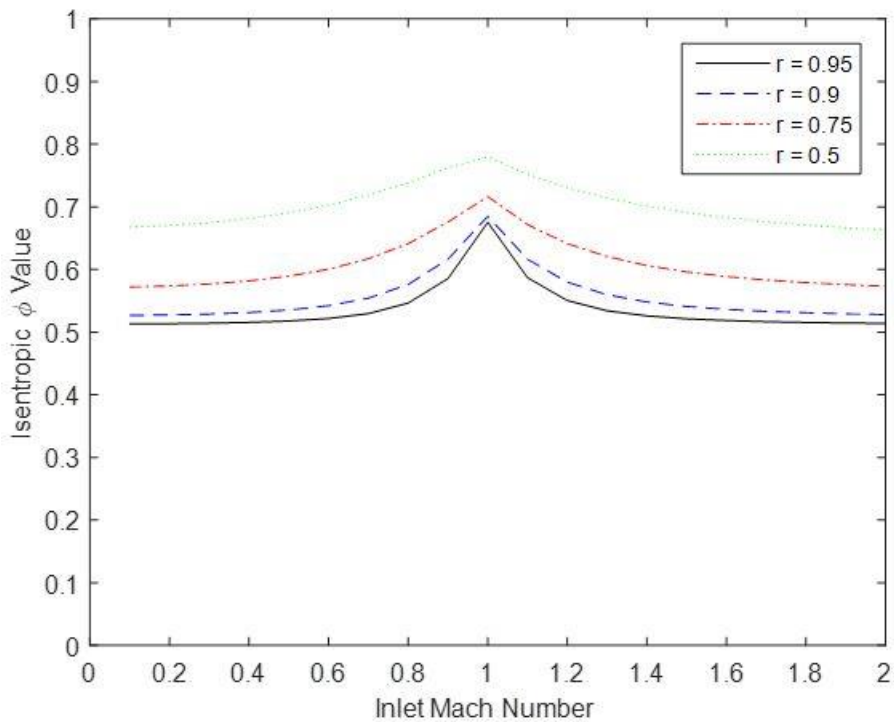

Figure 3.2: Graph of the *φ* Parameter Over Multiple Area Ratios and Inlet Mach Numbers

Figure 3.2 leads to some interesting results. Recall that the *φ* parameter is an average value of the side wall pressure scaled to be one half when the average pressure is halfway

[4] A table of all of the data used to make Figures 3.2 and 3.3 can be found in Appendix C

between the inlet and exit pressure. What Figure 3.2 shows is that the average side wall pressure for isentropic compressible flow is near 0.5 for low and high speed flows. As the flow approaches Mach one, however, the average side wall pressure increases dramatically. As the passageway expands more rapidly, a smaller *r* value, the φ value increases. Smaller *r* values also lead to a less pronounced increase in φ around Mach one is as well.

The next task is to investigate how the φ parameter varies with the generation of entropy. Let the area ratio be constant at 0.9, and then vary the amount of total pressure lost due to entropy generation. Figure 3.3 shows the resulting φ, from varying the inlet Mach number and the total pressure ratio.

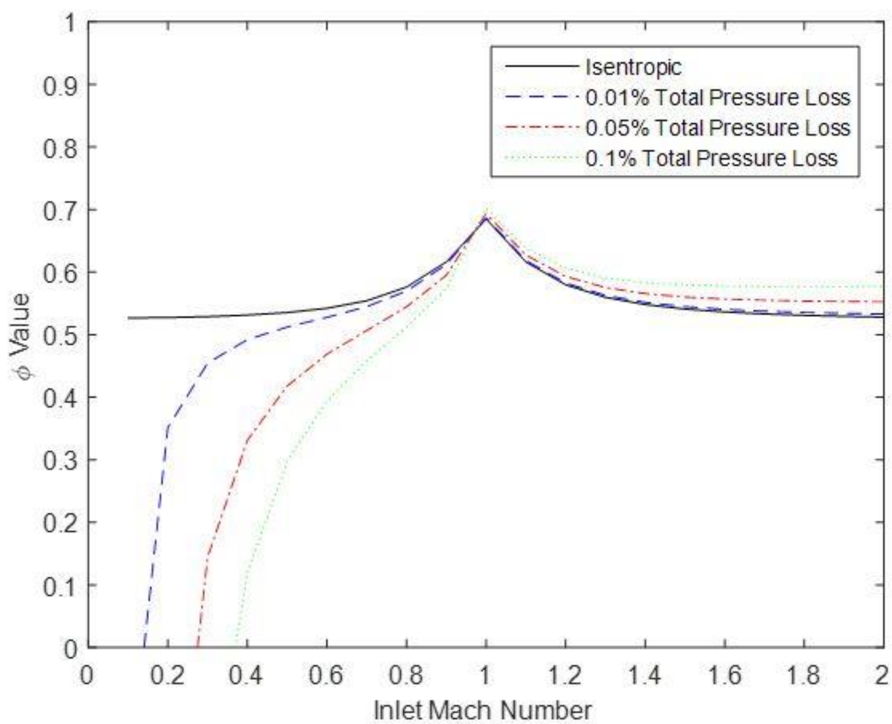

Figure 3.3: Graph of the φ Parameter Over Multiple Total Pressure Ratios and Inlet Mach Numbers

There are two very different system responses for the case where total pressure is decreased. For subsonic systems, the decrease in total pressure drastically lowers the φ value. These pressure losses eventually force the system to a φ of zero and below. As more total pressure is lost, the greater this effect. For supersonic flows, however, the total pressure loss increases the resulting φ. The increase in φ caused by total pressure loss is much less noticable compared to a change in area ratio, but is still prevalent. The shape of the curves for each total pressure ratio seems to maintain the same shape as the isentropic case, unlike the low area ratio curves seen in Figure 3.2.

4 Analysis of Shock-plugs in Converging-Diverging Micronozzles

4.1 Geometry and Dimensions of Micronozzle

The classic problem described in Figure 1.1, where the lowering of back pressure caused a shock wave, can be adapted to the shock-plug model. Two key facts provide a starting point for this adaptation. One, the shock wave is fixed once the back pressure is set. Two, the shock-plug model converges to the shock wave solution as the thickness goes to zero, or the area ratio goes to one. This means that as the back pressure decreases between the first and second critical pressures, a shock plug will form. This can be seen graphically in Figure 4.1.

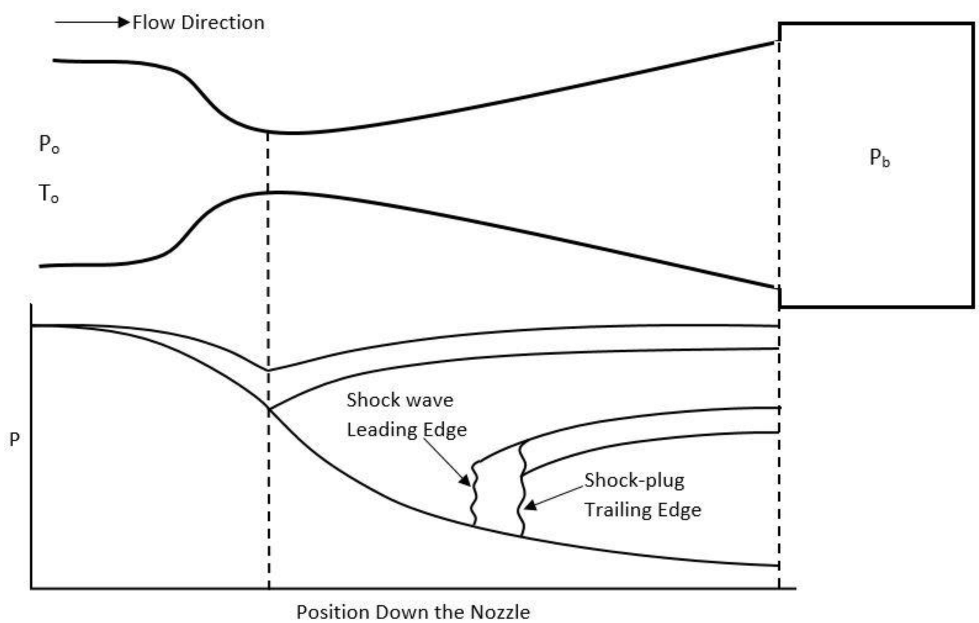

Figure 4.1: Effects of Back Pressure over a Converging-Diverging Nozzle with a Shock-plug Forming

This means that a nozzle could be designed in such a way that the shock thickness, as predicted by Granger [3], would lead to a significant area ratio, allowing for insight into the two additional parameters, r and φ.

The first step is to define a nozzle geometry and dimensions such that the shock thickness would lead to a non-negligible area ratio. Firstly, the general shape of the nozzle needs to be determined. Louisos goes into depth about micronozzle manufacturing [4]. Due to the process of micro-scale manufacturing, the nozzle would normally have to have a square cross section [4]. The nozzle will have a constant depth because of these manufacturing techniques. Using a micro-nozzle with a linearily diverging section, similar to one that was investigated by Louisos, results in a nozzle that further simplifies the parameterization and is realistic as well. Louisos performed simulations for a range of angles of the nozzle's side wall in the diverging section, from 10^o to 50^o. Above 30^o, the flow has a non-negligible transverse velocity [4]. Considering that the shock-plug model is only quasi-one-dimensional, a 15^o slope in the diverging section of the nozzle should be sufficient to allow for a realistic nozzle that does not have a large transverse flow component.

Since the nozzle must be choked, Mach one at the throat, to produce the shocks that are being analyized, the converging section of this nozzle is not important to this problem. The flow's properties are dependant on the total properties entering the nozzle and the area of the throat. The converging section simply does not effect the analysis, so long as it still meets the adiabatic and isentropic assumption.

The next concern is the size of the micronozzle's throat. For the thickness of the shock-plug to be of significant, the width of the nozzle needs to be at a similar length scale to the thickness. Using an example from Granger, a shock is approximately 30 nm thick. His example is for air at 1atm, and 100 ^{o}F flowing at 2000 ft/s. The viscosity is $1.8x10^{-4}$ ft^2/s, and the thickness is $9.0x10^{-8}$ ft [3]. This thickness converts to approximately 30 nm. From above, the nozzle for this problem is taken to have linear sidewalls that are expanding at 15 degrees. This means that a 30 nm axial length would create a 16 nm change in width of the nozzle. A 10% change in area, between the inlet and the exit of the shock, would cause a significant deviation from the standard shock wave model. This would mean that a shock with an inlet width of 160 nm would cause a situation where the standard shock wave model is not accurate. If the throat of the nozzle was set to 160 nm, as the shock-plug moved down the nozzle the difference in the areas would be less than 10 percent. To mitigate this, the throat of the nozzle for this problem will be 100 nm.[5] This allows for shock-plugs located throughout most of the nozzle's diverging section to have a signifigant area ratio. Figure 4.2 shows the diagram of this system.

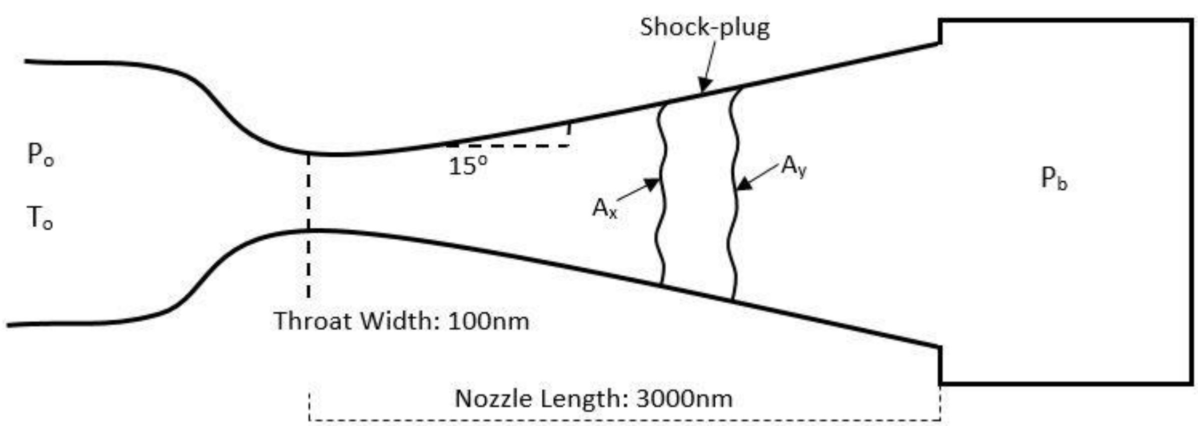

Figure 4.2: Dimensional Diagram of Model Micronozzle

[5] Code for this nozzle's geometry can be found in Appendix D.

The last part of this system is the initial flow parameters and assumptions. Classically, the inlet of the nozzle has fixed stagnation temperature and pressure. The assumptions used are that the working fluid is an ideal gas, the nozzle is isentropic throughout, excluding the shock, and that the flow is quasi-one dimensional. From there, the static pressure at the exit of the nozzle is all that is needed to determine the solution of a normal shock wave. This is because the properties across the shock wave are entirely determined by the inlet Mach number as stated prior. In the case of a converging-diverging nozzle, the flow constantly speeds up as the area increases past the throat. Recall, Figure 2.1 shows what occurs as the back pressure is dropped and the inlet parameters are held constant. These will not be enough to fully define the shock-plug model, as will be discussed later.

4.2 Shock Wave Location

Recall that if the back pressure of the system is between the first and second critical pressures, a normal shock wave will form. Since the shock wave is completely determined by the inlet Mach number, the exact location of this shock can be found using an iterative method. The distance of the shock from the throat of the nozzle is guessed. Since the flow is assumed to be isentropic throughout the nozzle, excluding the shock, the inlet Mach number of the shock can be found using the variable area relation as found in Anderson. These relations come from first principles in a quasi-one dimensional channel. The flow properties after the shock can be found using the normal shock relations. Then the variable area relations are used again to find the back pressure that would cause the shock wave to exist in that location. Once the calculated back pressure matches the one given by the problem, the location of the shock wave is found. This general process will be used to help limit the location of the shock-plug. The shock-plug has to converge to the shock wave solution as the area ratio goes to 1, so the location of that shock-plug must be dependent on the shock wave's location in some manner.

4.3 Relationships for Compressible Flow in a Nozzle

Section 2.3 discussed the relationships of the flow parameters across a shock plug. While these equations are useful for determining the characteristics of a shock-plug, they do not relate to the known parameters of this nozzle problem. In order to use the relations, they must be related to known quantities.

The first step is relating the inlet conditions of the shock-plug to the given total temperature and pressure. Since the nozzle is assumed to be isentropic, this calculation is done using standard isentropic flow relations. The location of the inlet of the shock-plug is crucial to this calculation. If the location of the leading edge of the shock-plug is known, or guessed, then the area of the nozzle at that location will determine the inlet flow conditions by use of the area Mach number relation [2].

$$\frac{A}{A^*} = \frac{1}{M^2}\left[\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M^2\right)\right]^{\gamma+1/\gamma-1} \tag{4.1}$$

where A^* is the area of the throat of the nozzle. It should be noted that this relation has two real solutions, one for subsonic and one for supersonic Mach numbers. This problem dictates that we take the supersonic solution as the inlet Mach number, otherwise no shock would form.

The next step in relating the known total properties to the shock-plug relations is to convert the total properties into static ones. With the inlet Mach number of the shock-plug guessed, and the total properties known, Equations 3.1 and 3.2 can be used to find the inlet pressure and temperature of the shock plug. The equations are reproduced below.

$$\frac{P_o}{P} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\gamma / \gamma - 1} \tag{3.1}$$

$$\frac{T_o}{T} = 1 + \frac{\gamma - 1}{2} M^2 \tag{3.2}$$

Equations 4.1, 3.1, and 3.2 are all that is needed to relate the known properties of the nozzle to the inlet conditions of the shock-plug.

The shock-plug relations can now be used to solve for the change in flow properties across the shock-plug. It should be noted that the parameters φ and r are not constrained by the three shock-plug relations. This means that they must be chosen by different means. For instance, r is determined by selecting the inlet location and the thickness of the shock-plug. For now, they can be considered given.

Once the static flow properties at the exit of the shock-plug are known, Equations 3.1 and 3.2 can be used to find the total properties after the shock-plug. This means that the back pressure can be found by application of the continuity equation. Other properties, like side wall pressure, can also be calculated once the inlet and exit conditions of the shock-plug are known. [6]

4.4 Closing the Loop on the Shock-plug

The primary problem with the addition of the area ratio in the shock model is that the parameters r and φ are introduced. Without additional relations the shock-plug problem is not

[6] The MATLAB code implimentation of this solution method is shown in Appendix E

fully constrained. Since two parameters were added, there needs to be two additional constraints. Entropy generation is a typical constraint used to solve a compressible flow problem, but this leaves one more constraint to be desired. The last constraint that will be implemented on this model is location. As discussed previously, the shock-plug's location is related to the shock wave's location because the two solutions converge as the area ratio becomes 1. This model will limit the location of the shock-plug to either begin or end at the shock wave's location. The two cases of this constraint are shown below in Figures 4.3 and 4.4.

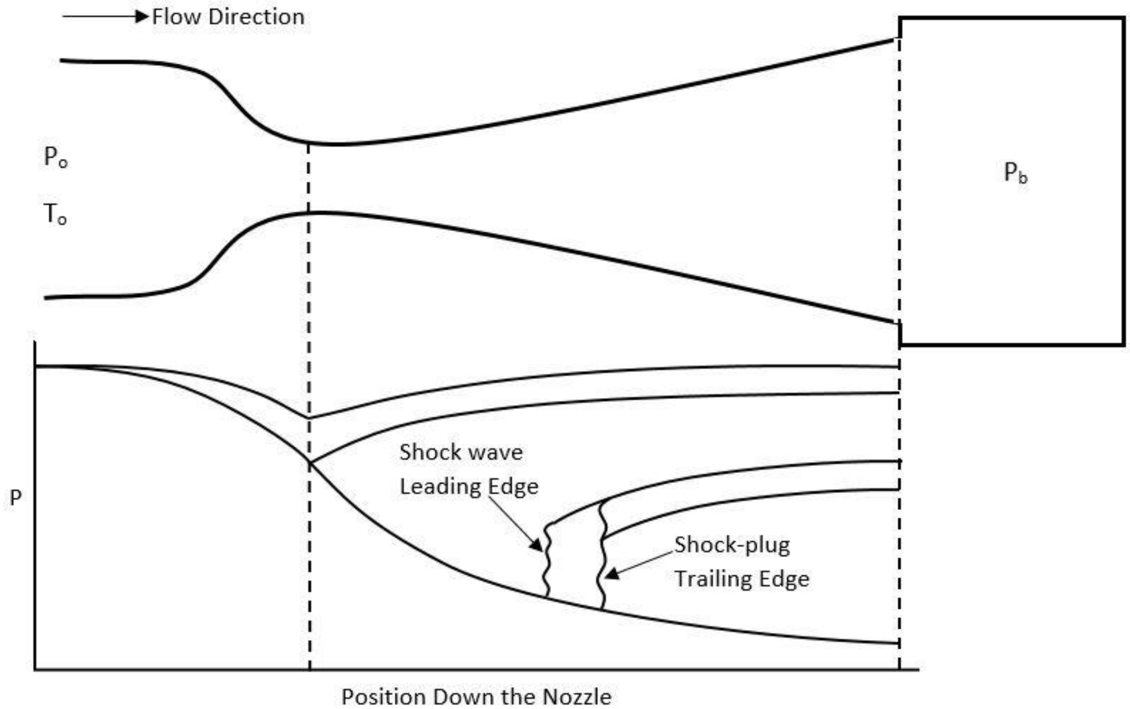

Figure 4.3: Shock-plug with Shock Wave Location as Leading Edge

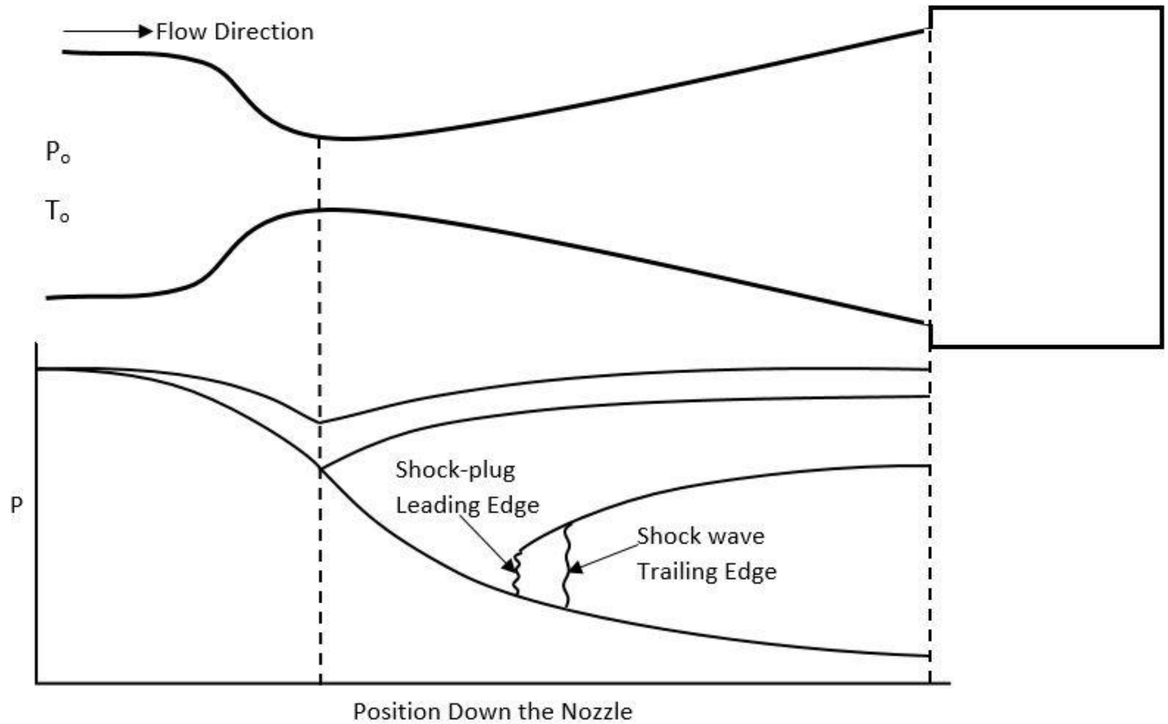

Figure 4.4: Shock-plug with Shock Wave Location as Trailing Edge

Since systems tend towards maximum entropy generation, a first approach would be to pick one of the parameters, *r* and *φ*, so that entropy is maximized. Figure 2.6 shows that a decrease in the area ratio, an increase in shock thickness, will create more entropy at any given inlet Mach number. The shock-plug can continue to expand throught the nozzle until it fills the system completely because there is not inherent size restriction from the first prinicples. This means that the enttopy generation does not have an upper limit by varying area ratio; making it a bad candidate to set to create a maximum entropy.

The *φ* parameter, on the other hand, is a optimal candidate for setting to one value. Figure 2.10 shows that the entropy generated by the shock-plug increases as the *φ* value decreases into the negatives. *φ* does have a lower bound unlike the area ratio. At any given inlet Mach number there exists a value for *φ* that creates a side wall pressure of zero. Any *φ* less than this is unrealizable. Since the maximum entropy is generated at the most negative *φ* value, a side pressure of zero should result in the maximum entropy gain for any given inlet Mach number and area ratio.[7]

The other option to determining the solution for the shock-plug is to simply chose one of the values of thickness or *φ* arbitrarily, and find the other value to meet the system requirements. For this method, the thickness of the shock-plug will be set to a range of values, and the *φ* will be calculated. This allows for an insight into how this model behaves with different inputs, instead of just one fixed *φ* value. This method also determines if there is only one solution for any given nozzle, or if multiple combinations of thickness and *φ* can produce the same results.

4.5 Effects of Parameters on Shock-plugs

The system in Figure 4.2 with the constraints discussed in Section 4.3 allows for the effects of *φ* was well as *r* on the flow parameters can be investigated. To better determine the effects of the paramters *φ* and *r*, the shock-plugs' leading edge is placed at a single location. This is shown in Figure 4.2 where a decrease in *r*, an increase in shock-plug thickness, would move the shock-plug trailing edge closer to the exit of the nozzle. This ensures that the inlet conditions are consistent across all of the shock-plugs. From there, the parameters, *φ* and *r*, are varied over a wide range. For this setion, *φ* is varied from -1.0 to 3.4 and the thickness of the shock-plug is

[7] This solution method varies from the one with a given φ values due to the lack of the φ parameter, so it has its own code, which is shown in Appendix E after the first method.

varied from 0 to 100 nm. The exit conditions, side pressure and entropy generation are then calculated and compared.[8]

How the parameters, φ and r, effect the exit flow properties: Mach number, pressure, and temperature is shown in Figures 4.5, 5.6, and 5.7. An area ratio of one leads to the same result as a shock wave independent of and value for φ. As the area ratio becomes smaller, the deviation from the shock wave solution increases. The effects of φ becomes more pronounced as the area ratio decreases from the shock wave solution at one.

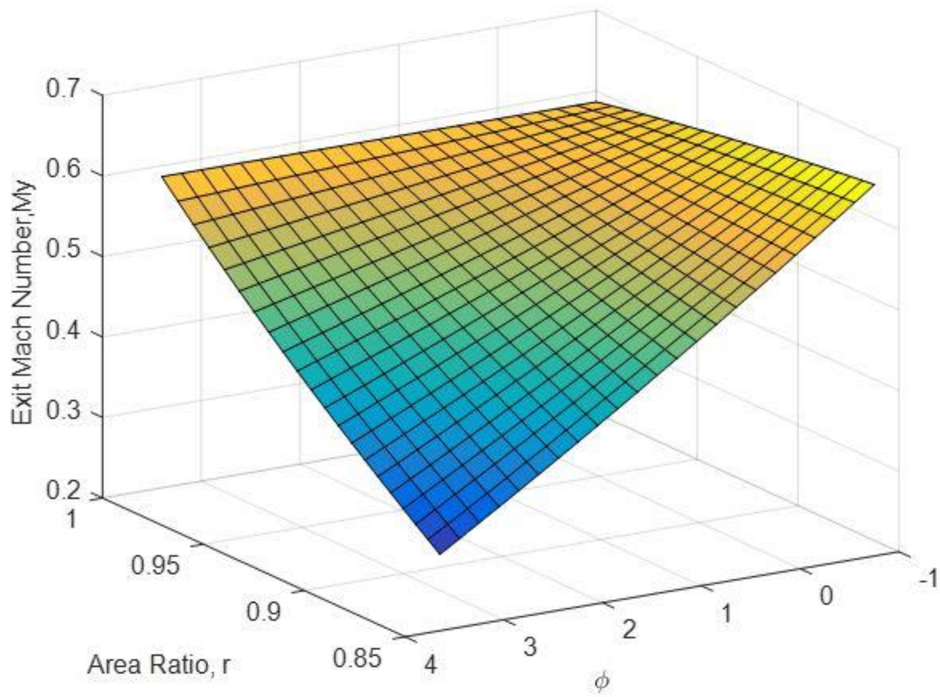

Figure 4.5: Effects of φ and r on Exit Mach Number in a Shock-plug

The exit Mach number increases as φ becomes more negative. As φ increases, the exit Mach number decreases rapidly.

[8] The MATLAB code used to impliment this method is found in Appendix F as well as contour plots.

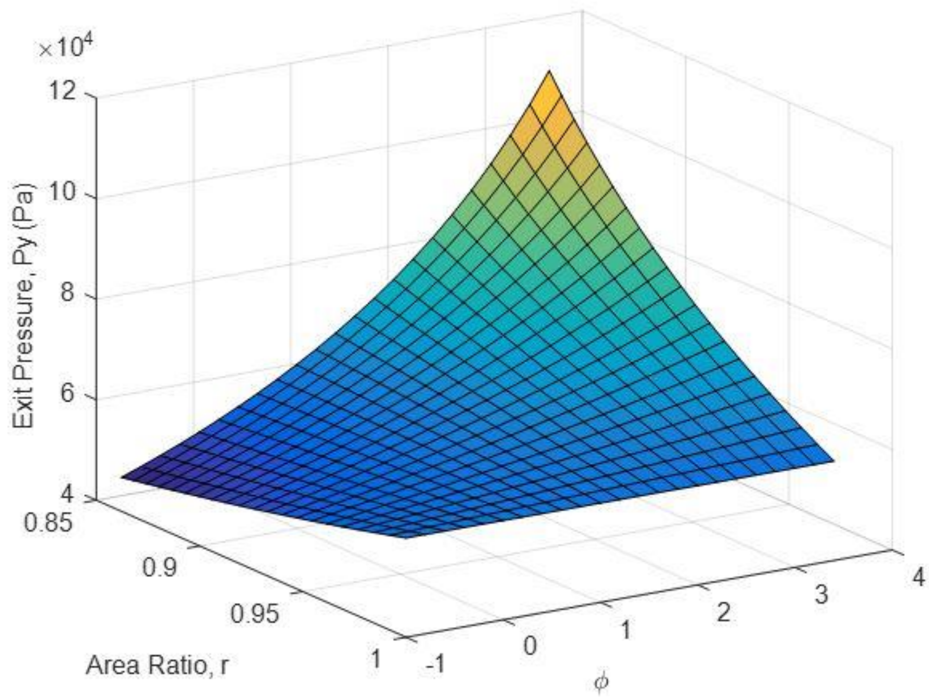

Figure 4.6: Effects of *φ* and *r* on Exit Pressure in a Shock-plug

The exit pressure rapidly increases as φ increases, and slowly decreases as φ becomes negative.

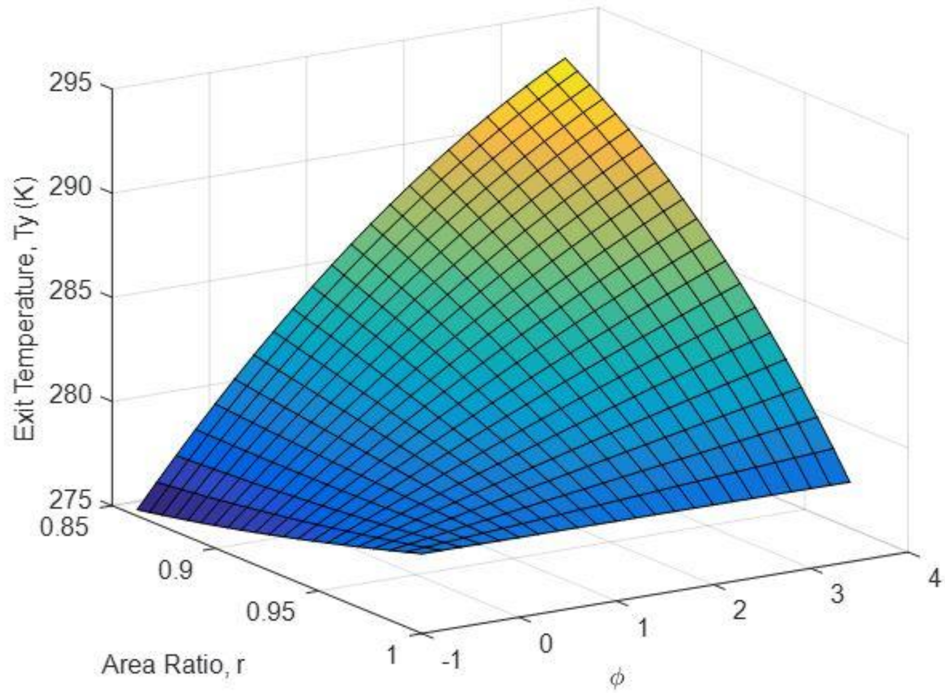

Figure 4.7: Effects of *φ* and *r* on Exit Temperature in a Shock-plug

Similar to the pressure, the temperature increases as φ increases and decrease as φ decrease.

As the flow properties change, back pressure also changes. Since the nozzle geometry is fixed for this problem, the back pressure is dependent upon the exit Mach number, exit pressure and the location of the trailing edge of the shock-plug. Figure 4.8 shows the back pressure caused by these changes. As with the exit properties, the back pressure matches the shock wave solution at an area ratio of one, but the effects of φ become pronounced as the area ratio decreases.

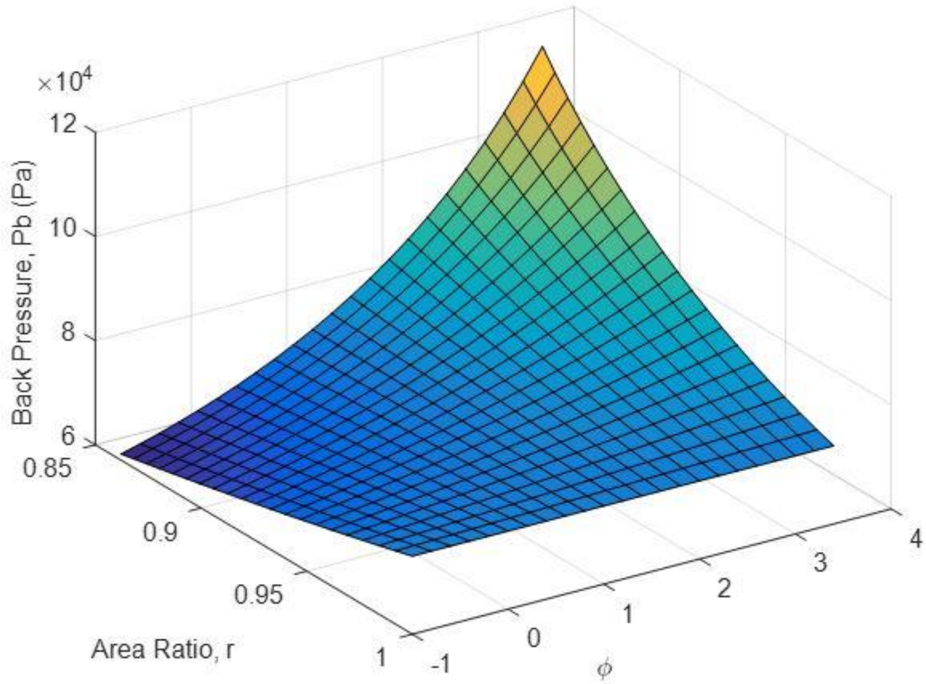

Figure 4.8: Effects of *φ* and *r* on Back Pressure in a Shock-plug

Figure 4.9 shows the entopy generation across this range of paramters and the first limits of a shock-plug. As the area ratio decrease and φ increases, the entropy generation decreases until it is negative. This is not realizable and shows that there is a combination of r and φ that will not produce a real solution. The red line on the surface in Figure 4.9 shows where the shock-plug produces zero entropy. Below that red line are unrealizable solutions. Entropy generation increases as φ decreases, however, leading to no lower limit to the φ value or and bounds on area ratio.

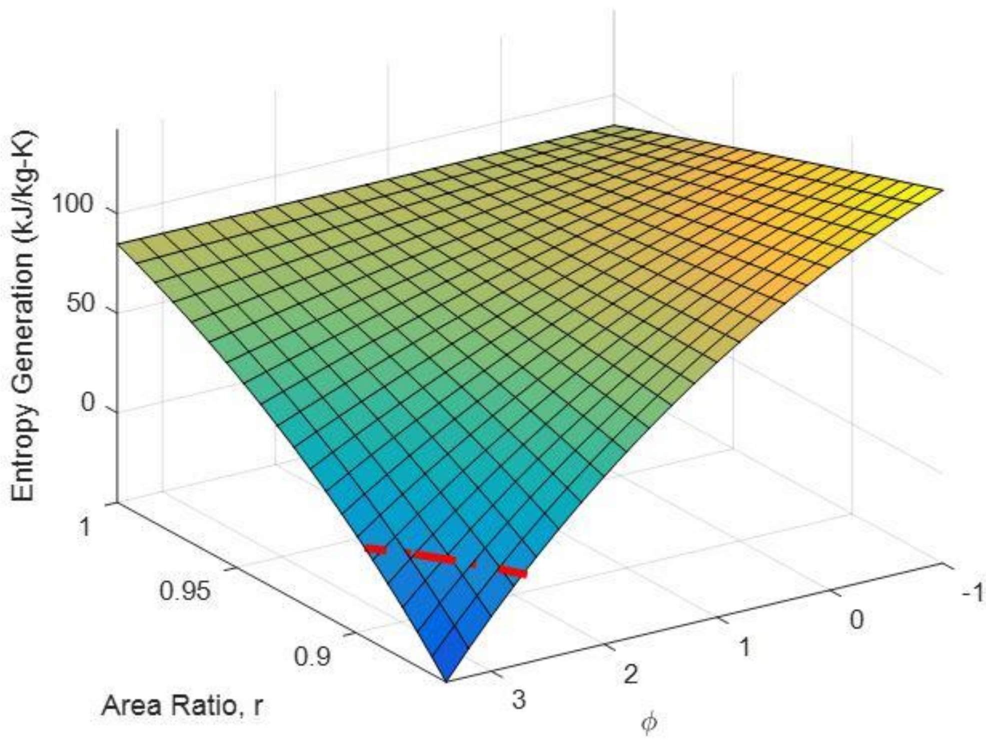

Figure 4.9: Effects of φ and *r* on Entropy Generation in a Shock-plug

Similar to the investgation in Section 2, the pressure on the side wall inside of the shock-plug is an important parameter to investigate. . Figure 4.10 shows how the side pressure changes with φ and r. The pressure on the side walls provides another limiting case for the shock-plug. This pressure cannot be below zero, otherwise it is not a realizable system. Unlike the other parameters, side pressure is largely a function of φ. This is because the shock wave solution has no side pressure, and therefor the $r=1$ case does not converge to a single solution. As the area ratio becomes small the effects of φ become more nonlinear.

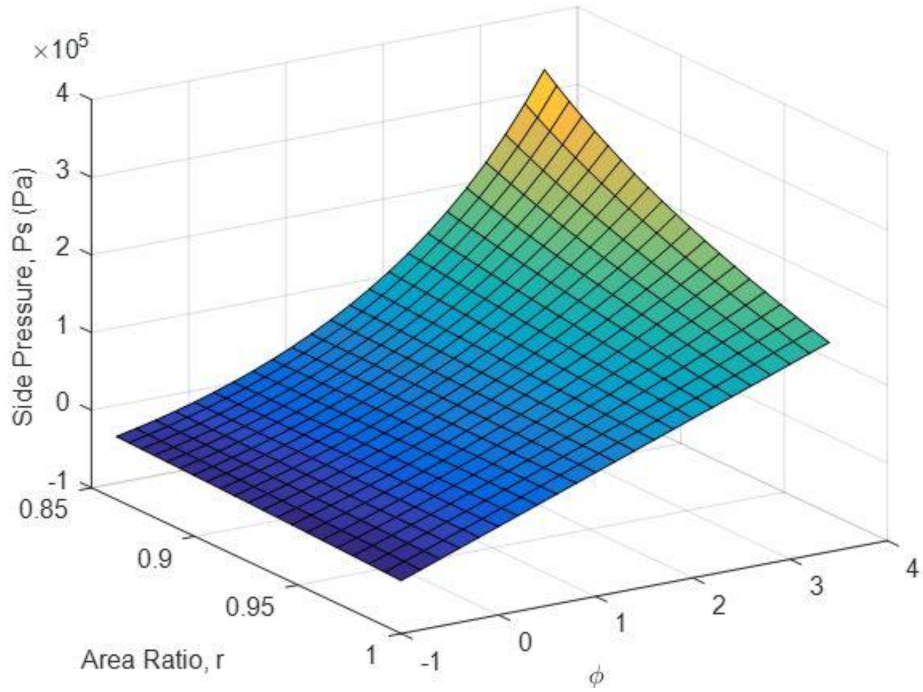

Figure 4.10: Effects of φ and r on Side Pressure in a Shock-plug

4.6 Results of Maximizing Entropy

Section 4.4 discussed limiting the positions of the shock-plug to either beginning or ending with the shock wave's location to limit the possible valid solutions. The first approach to create a single solution was to maximize entropy by setting the side pressure to zero. Meaning that the thickness determines all of the exit properties. For Figures 4.11 and 4.12 the shock-plug's leading edge is the same as the shock wave solution, similar to what is shown in Figure 4.3. Figure 4.11 shows the entropy generated with varying thicknesses with the leading edge of the shock-plug at the same location as the shock wave. The horizontal line is the entropy generated from a shock wave the satisfies the back pressure constraint.

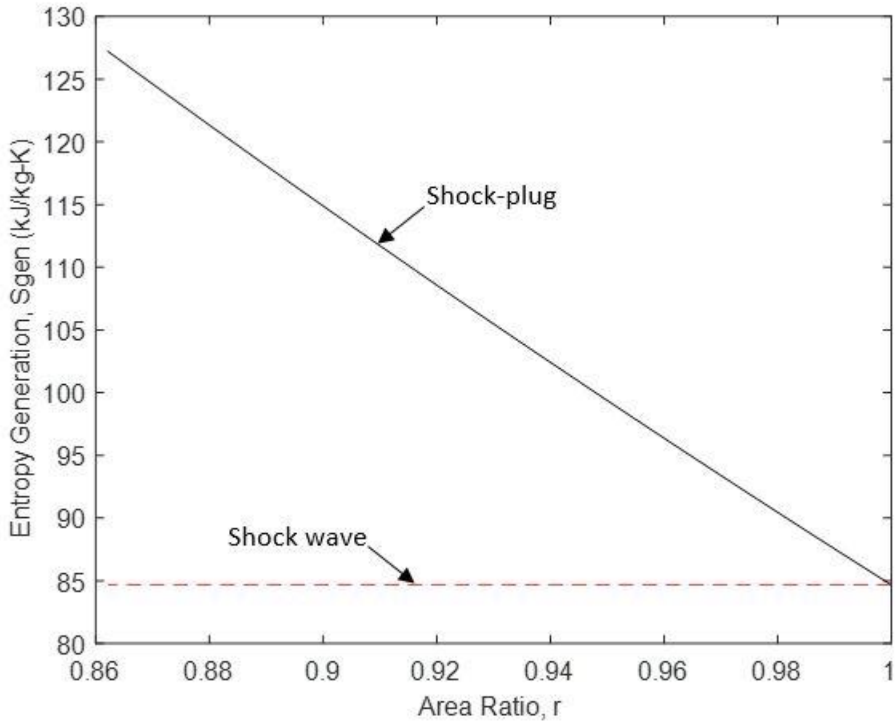

Figure 4.11: Maximum Entropy Generated by a Shock-plug with the Shock Wave Position as the Leading Edge

The only point where the shock wave and shock-plug entropy generated matches is when the area ratio equals one. This is the shock wave solution. Figure 4.12 shows the back pressure for the same shock-plug set up. Again the shock-plug's solution only matches at an area ratio of one.

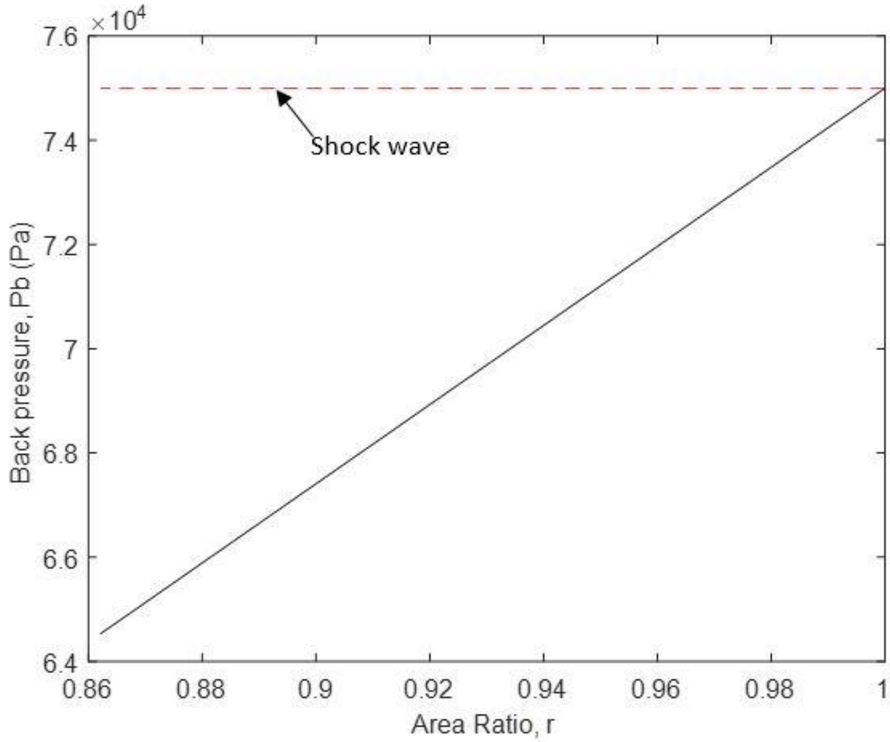

Figure 4.12: Back Pressure of a Maximum Entropy Generation Shock-plug with the Shock Wave Position as the Leading Edge

For Figures 4.13 and 4.14 the shock-plug's trialing edge is the same as the shock wave location as shown in Figure 4.4. Even with the trailing edge of the shock-plug is set to the shock wave location, and entropy generation is still set to a maximum, the results remain the same. The only area ratio when the two solutions match is at an area ratio of one. Figure 4.13 and Figure 4.14 show the entropy generated and back pressure of this case, respectively.

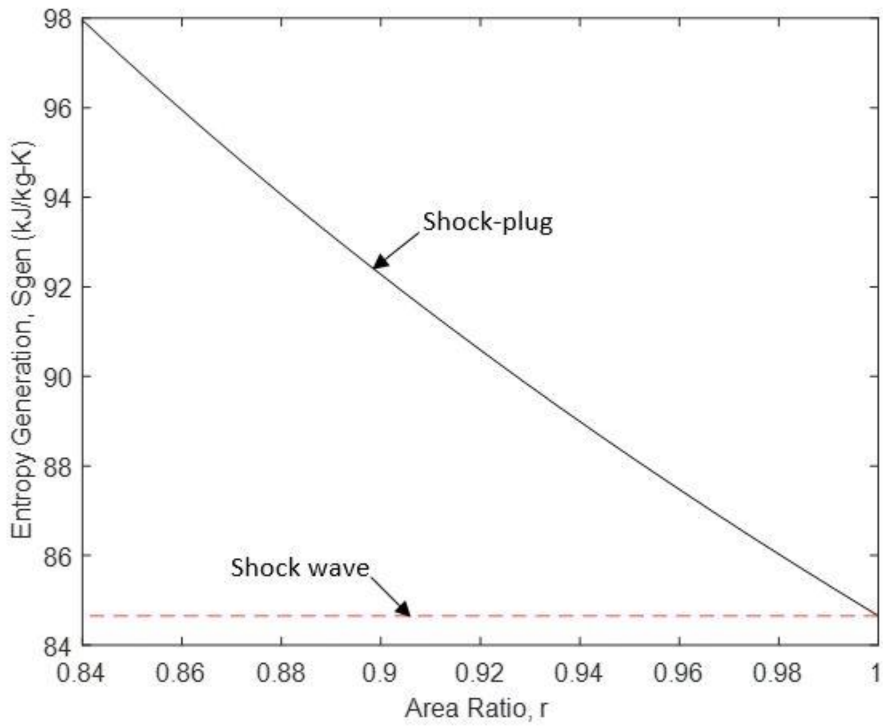

Figure 4.13: Maximum Entropy Generated by a Shock-plug with the Shock Wave Position as the Trailing Edge

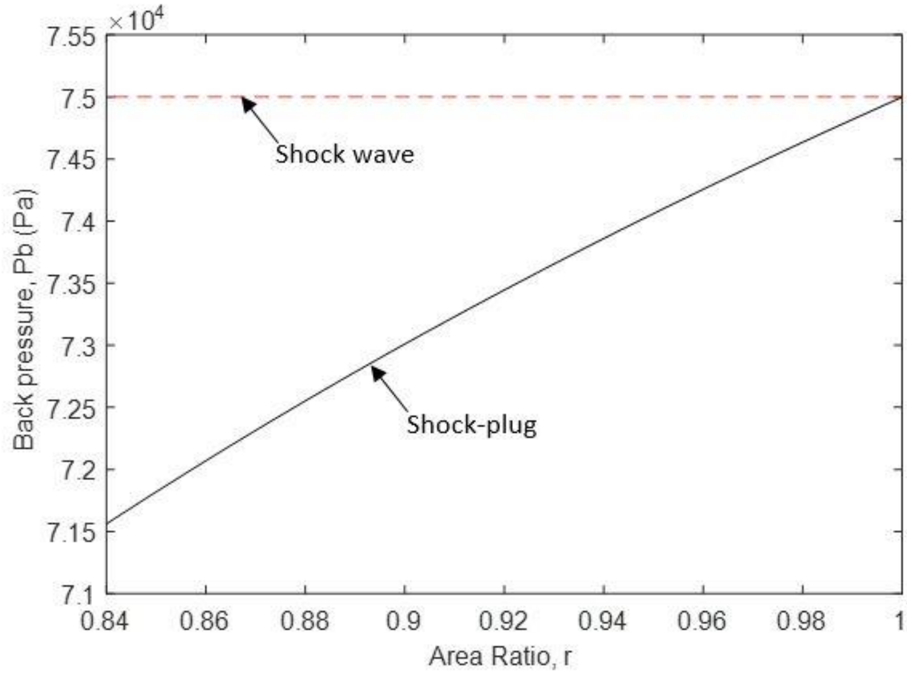

Figure 4.14: Back Pressure of a Maximum Entropy Generation Shock-plug with the Shock Wave Position as the Trailing Edge

4.7 Results of Varying φ and r to Match Back Pressure

The second method was to set the back pressure and vary the area ratio, and φ parameter until the back pressure was the set value. From there, the shock plug(s) with the same entropy generation as the shock wave can be considered valid solutions. Since the previous sections shows that there is not way to match both the back pressure and entropy generation for a specific case of φ an area ratio, outside of the shock wave solution, only the case where the shock-plug's trailing edge is at the shock wave location will be considered. Since the shock-plug is expanding towards the nozzle's throat and the isentropic solution after the shock-plug is deterministic, the exit flow properties, Mach number, pressure, and temperature, are all constant. This means that as the area ratio gets smaller the inlet conditions change. Figure 4.15 shows how the inlet Mach number changes as the area ratio decreases.

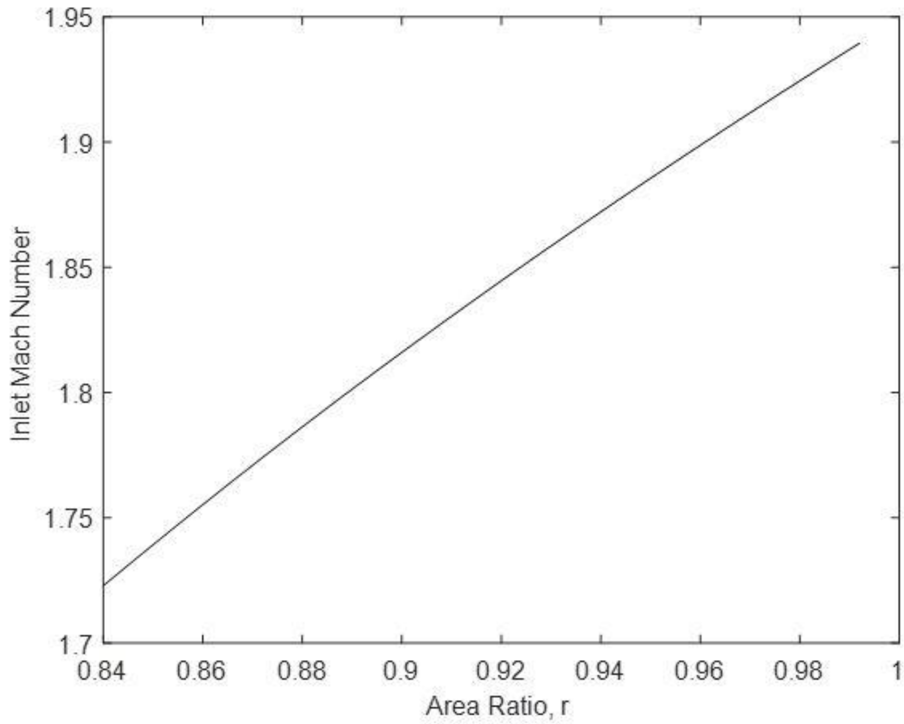

Figure 4.15: Inlet Mach Number of Matched Back Pressure Shock-plugs with the Shock Wave Position as the Trailing Edge

The resulting value for φ that produces the correct back pressure is shown in Figure 4.16. The value for φ that produces a zero side wall pressure is also shown as a way to tell if the value of φ is a realizable. It should be noted that all of the values of φ that match the back pressure are below zero. Recall that Equation 2.13 is used to define φ as

$$P_s = P_x + \varphi\left(P_y - P_x\right) \tag{2.13}$$

And that a value of below zero means that the side pressure is below the inlet pressure. This would mean the pressure inside of a shock-plug is a partial vacuum.

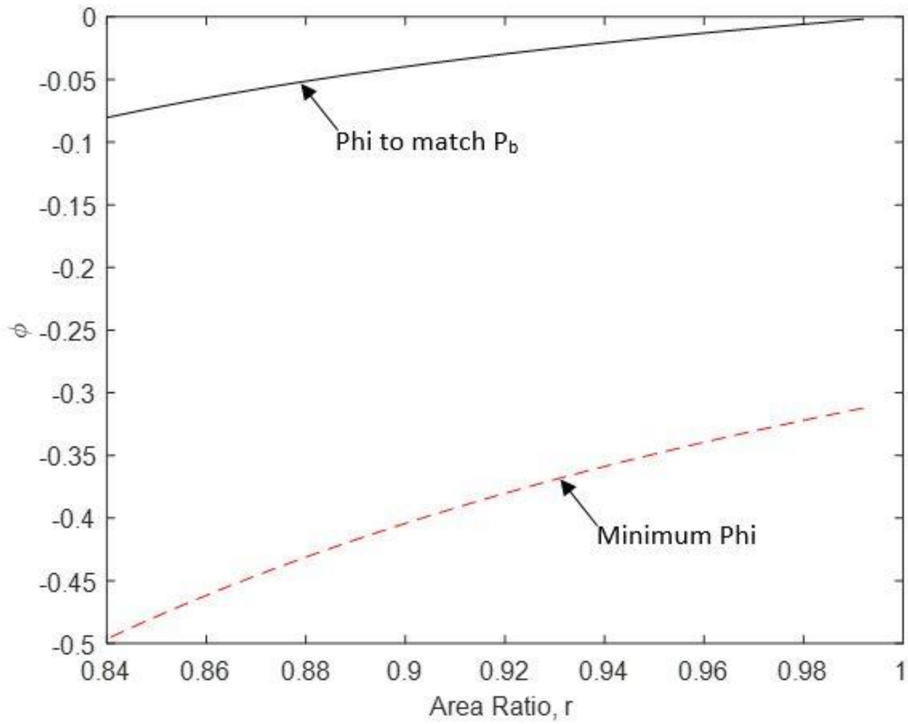

Figure 4.16: Values for φ of Matched Back Pressure Shock-plugs with the Shock Wave Position as the Trailing Edge

These values for φ all generated shock-plugs that meet the back pressure constraint. This means that for every area ratio, there is one value for φ that will create a given back pressure. Figure 5.13 shows the entropy generated by these shock-plugs.

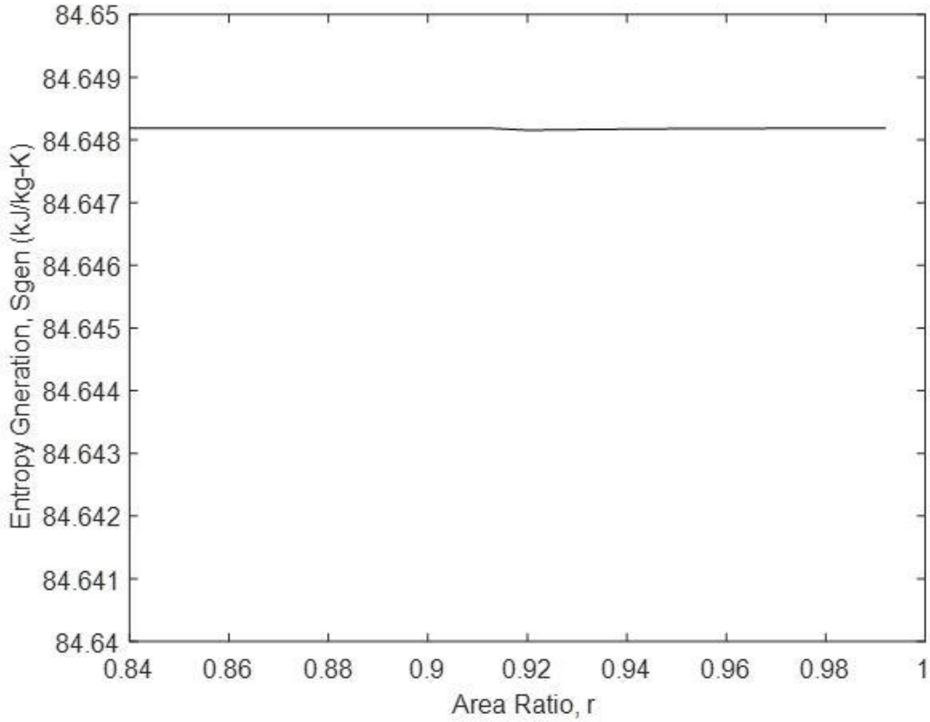

Figure 4.17: Entropy Generated by Matched Back Pressure of a Shock-plug with the Shock Wave Position as the Trailing Edge

Similar to the exit conditions, the entropy generated by these shock plugs are all the same, and all equal to the shock wave solution. The minor discrepancy in the line is a value of about 0.1 J, which is inside of the precision error of the code. By the description of this method, this means every thickness of shock-plug creates one solution that meets the back pressure constraint and produces the same entropy as a shock wave with the same back pressure constraint.

5 Conclusion

5.1 Direct Shock-plug Conclusions

From the Figures shown in Section 4.6, a shock-plug can produce a given back pressure if the trailing edge of the shock-plug is at the location where the infinitesimally thin shock wave is located. By forcing those two parameters the entropy generation of the shock-plug also matches that of the shock wave. By considering Figure 5.17, a shock-plug will match the shock wave solution's entropy generation from $r = 0.84$ to $r = 1.0$. Since the entropy generation is effectively constant within the range modeled, any shock-plug area ratio can match the back pressure constraint at a certain value for φ. Figure 4.16 shows that the slope of the minimum φ value is steeper than that of the actual φ value that matches the back pressure, meaning that the value for φ will not reach a condition where the side pressure is zero. This may change at extremely low area ratios.

Since effectively any area ratio will lead to a valid solution, the thickness of the shock-plug does not appear to be constrained by the first principles under the assumptions made. There must be some other effects that limits the size of the shock-plug that is not captured with a quasi-one-dimensional invisid model.

There is an interesting conclusion caused by the solution method used to create the results in Figure 4.15 through 4.17. Setting the back pressure of the nozzle and only allowing the shock-plug's leading edge to advance up-stream as the area ratio decreases fixes all of the exit conditions of the shock-plug. Figure 5.1 shows this graphically.

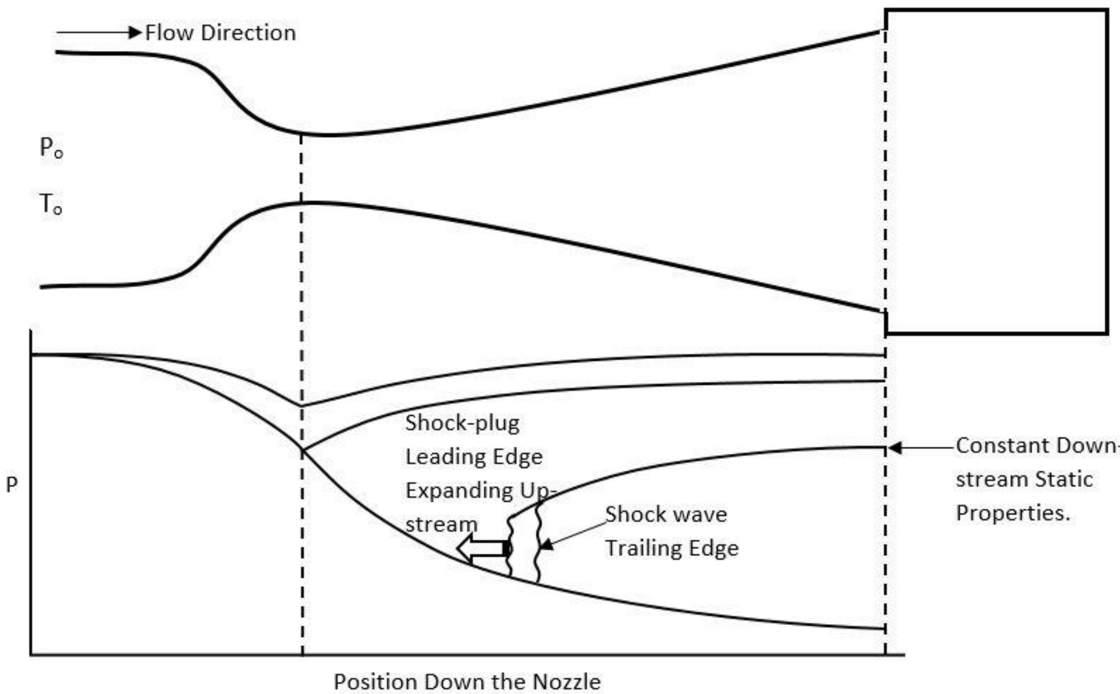

Figure 5.1: Constant Exit Conditions from Shock-plug Leading Edge Variation

The exit of the shock-plug is set to be the same area and the back pressure is held constant. Recall that Equation 3.1 relates the pressure at any given point to the Mach number and total pressure of the flow. This, paired with Equation 4.1, the relation to area and Mach number, leads to a single possible solution for the exit condtions of the shock-plug. What this results in is that the shoc-plug can expand forward, with ever decreasing values for r and there will be a value for φ to match the exit conditions.

Back pressure is also set as a constraint in this problem. Recall that Equations 3.1 relates the pressure to the total pressure to the Mach number at that location. This means that the back pressure is constrained by the total pressure after the shock-plug and the exit Mach number. The shock-plug, by virtue of entropy generation, changes the total pressure so that it not the same after the shock as it was before. There is however a specfic temperature at the exit of the nozzle, a back temeperature. This temperature can be related to its total state and the Mach number at the exit as well. Recall Equation 3.2. Unlike the pressure, the total temperature does not change across a shock. This means that the picking a back pressure set a back temperature, and this in turn constrains the conditions that exit the shock-plug.

Due to the fact that this model fixed the exit conditions of the shock-plug, a system that only measured the inlet and exit conditions of the nozzle would not be able to discern the difference the size of the shock. If the side wall pressure, on the other hand, was measured at various points along the micro-nozzle, the shock-plug would appear to be a drop in pressure between two different points. The parameter φ allows the side pressure to be scaled to easily comparable values. Since the nozzle is assumed to be isentropic throughout, except at the shock-plug, Figure 3.2 shows expected values for φ when there is not a shock present. These expected values roughly range from 0.5 to 0.8. Figure 4.16, on the other hand, shows what values of φ are needed to meet the back pressure constraint in the nozzle. These values reach a maximum near zero and decrease as the area ratio of the shock-plug decrease. This result means that a plot of local averages in the side pressure of the nozzle will be a curve around 0.65 until the shock-plug forms. Then the local average would drop to below zero inside of the shock-plug before increasing back up to around 0.65. Figure 5.2 shows what the value of φ could look like as in respect to position in the nozzle. The φ should rebound to almost the same value as before the shock-plug due to the symmetry present in Figure 3.2.

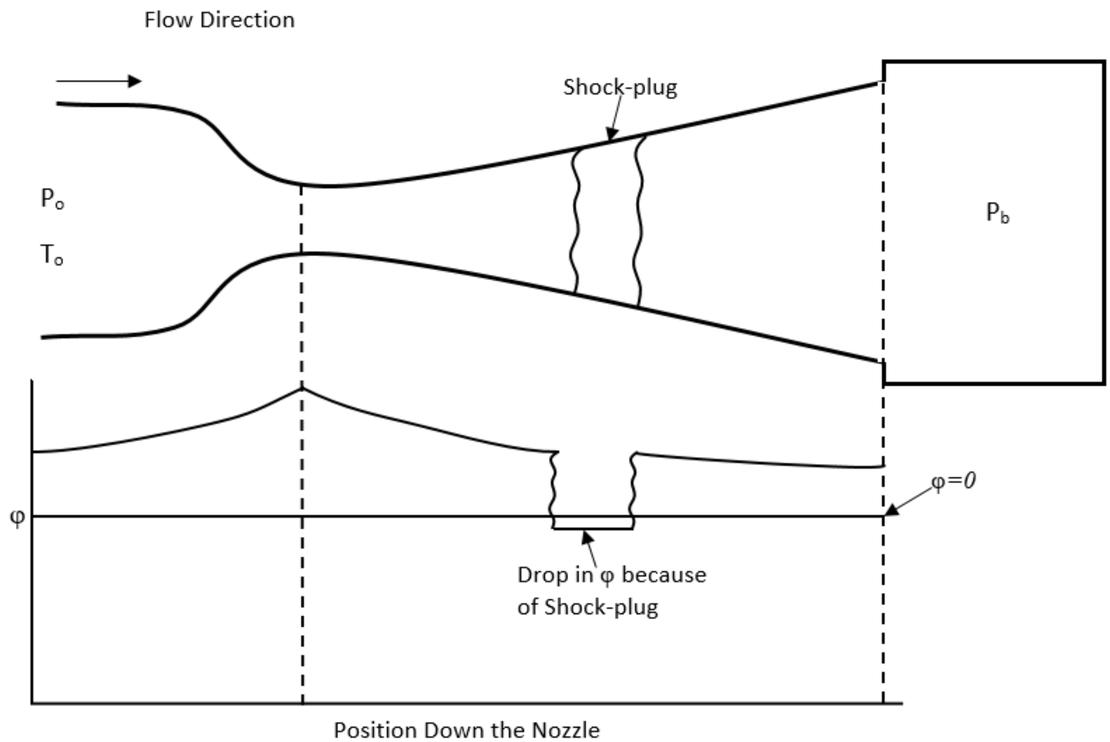

Figure 5.2: Approximation of *φ* with Respect to Position in the Nozzle

Since *φ* is related to the side pressure, a plot of the pressure of the fluid as it moves down the nozzle can be produced. This is due to the quasi-one-dimensional assumption. If the pressure is only a function of location in the flow direction, then the pressure at the side wall would also be the pressure of the fluid at that same distance from the throat. Figure 5.3 shows what the pressure of the flow is as the flow moves through the nozzle, according to this model.

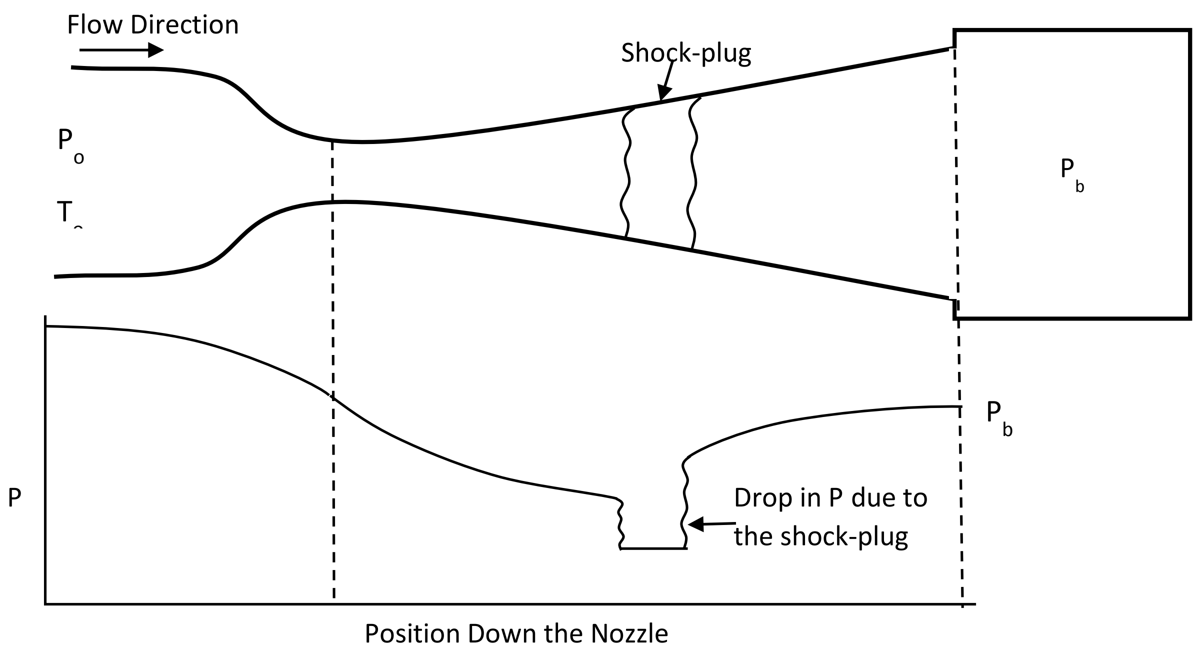

Figure 5.3: Approximation of Pressure with Resptect to Position in the Nozzle

5.2 What Shock-plugs Say about Shock Waves

One of the most profound results from the this model is how the local average side wall pressure, φ, drops below zero to create the same results as a shock wave. The corresponds to a side pressure of below P_x and P_y. While this could be useful to identify the thickness of a shock-plug experimentally, it also provides insight into what creates entropy in a shock wave. The common convention is that the entropy generated be a shock wave is the result of thermal and viscous effects in the fluid. Considering that the shock wave happens too quickly for thermal effects (the reasoning behind calling a shock wave adiabatic) and that the model shown in Anderson is invicid this does not hold up. The shock wave model does not take into consideration thermal or viscous effects, but it still produces entropy. There must be something in the shock wave model that produces entropy while keeping the assumptions the same.

From Figure 2.10, the entropy generated by a shock-plug increases as φ decreases. As a reminder, φ is part of the average of the side pressure as scaled to the inlet and exit pressures of the system, the shock-plug in this case.

$$P_s = P_x + \varphi\left(P_y - P_x\right) \tag{2.13}$$

A value of φ below zero means that the side pressure is below the inlet pressure of the shock-plug. The assumption that this system is quasi-one-dimensional means that the pressure inside of the shock-plug is the same as the side wall pressire and would also be below the inlet and exit pressure. For this to be the case, there has to be a partial vacuum inside of the shock-plug. This partial vacuum means that the gas in the shock-plug is “expanding against a nothing.” Even though the flow recompresses on the other side of the shock-plug, it does not go through the decompression-compression cycle reversibly. It still creates entropy, as shown by the drop in total pressure.

The free expansion resulting from these negative φ values, would be what causes the entropy to be generated in a shock-plug. Since the shock-plug model does in fact converge to the shock wave solution as the area ratio becomes one, this means that the entropy produced by a shock wave could also be the result of free expansion and not thermal or viscous effects. This is reinforced by Figure 4.9 where decreasing the value for φ results in a higher entropy generation at all area ratios below one.

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APPENDICES

Appendix A

Appendix A shows the full derivation of the shock-plug relations. There are a few relations that will be called upon repeatedly. The first is change from velocity to Mach number. The two terms for this are definition of Mach number, $M = v/a$ and the speed of sound for an ideal gas, $a = \sqrt{\gamma RT}$. By substituting the speed of sound into the definition of the Mach number and solving for *v*, the first key relation is $v = M\sqrt{\gamma RT}$. The second key relation is the ideal gas model, $\rho = P/RT$. The conservation principles are applied across the shock-plug system as defined by Figure 2.2, which is reproduced below.

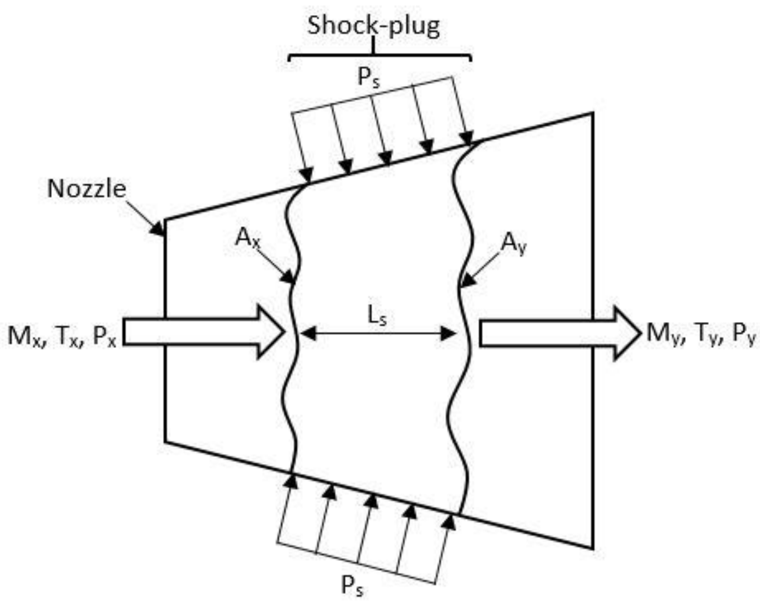

Figure 2.2: Schematic Diagram of Shock-plug Model

This system is assumed to be continuous, quasi-one-dimensional, steady-state, and, adiabatic. The working fluid is assumed to be an ideal gas with constant specific heats.

Conservation of mass is given as

$$\frac{dm}{dt} = \sum \dot{m}_{in} - \sum \dot{m}_{out} \ . \tag{A.1}$$

The model is assumed to be steady-state and there is only one inlet and exit Equation A.1 can be simplified to

$$\dot{m}_x = \dot{m}_y \ . \tag{A.2}$$

The mass flow of the inlet and exit flows can be shown as $\dot{m} = \rho v A$. Substituting this into Equation A.2 leads to the form of the conservation of mass for this model.

$$\rho_x v_x A_x = \rho_y v_y A_y \tag{2.1}$$

Substituting the first key relation into Equation 2.1 leads to

$$\rho_x A_x M_x \sqrt{\gamma R T_x} = \rho_y A_y M_y \sqrt{\gamma R T_y} \ . \tag{A.3}$$

Substituting the ideal gas model into Equation A.4 produces

$$\frac{P_x}{RT_x} A_x M_x \sqrt{\gamma R T_x} = \frac{P_y}{RT_y} A_y M_y \sqrt{\gamma R T_y} \ . \tag{A.4}$$

Combine like terms.

$$P_x A_x M_x \sqrt{\frac{\gamma}{RT_x}} = P_y A_y M_y \sqrt{\frac{\gamma}{RT_y}} \tag{A.5}$$

γ and R are both constants since the working fluid is assumed to have constant specific heats and appear on both sides of Equation A.6. Simplifying this leads to

$$P_xA_xM_x\sqrt{\frac{1}{T_x}} = P_yA_yM_y\sqrt{\frac{1}{T_y}} \tag{A.6}$$

Solving Equation A.7 for M_y/M_x creates Equation 2.9.

$$\frac{M_y}{M_x} = \frac{A_x}{A_y}\frac{P_x}{P_y}\sqrt{\frac{T_y}{T_x}} \tag{2.9}$$

The variable r is defined as $r = A_x/A_y$. This can be directly substituted into Equation 2.9 to get Equation 2.14.

$$\frac{M_y}{M_x} = r\frac{P_x}{P_y}\sqrt{\frac{T_y}{T_x}} \tag{2.14}$$

The conservation of linear momentum is given as

$$\frac{dp}{dt} = \sum F + \sum \dot{m}v_{in} - \sum \dot{m}v_{out}\,. \tag{A.7}$$

The force terms acting on this system are the pressure of the flow across the inlet and exit, as well as the projection of the side wall pressure in the flow direction. Again, the system is steady-state and only has one inlet an exit. Substituting these relations into Equation A.8 produces

$$0 = P_xA_x - P_yA_y + P_s\left(A_y - A_x\right) + \dot{m}v_x - \dot{m}v_y\,. \tag{A.8}$$

Reorganize the terms so that the inlet terms are on one side and exit terms on the other. This becomes Equation 2.10.

$$P_xA_x + \rho_x v_x^2 A_x + P_s\left(A_y - A_x\right) = P_yA_y + \rho_y v_y^2 A_y \tag{2.10}$$

The $\dot{m}v$ terms will need to be put in terms of Mach number. Using the relation for $\dot{m}$ used to get equation 2.1 is used again in the $\dot{m}v$ terms. The substitution results in

$$P_xA_x + \rho_xA_xv_x{}^2 + P_s\left(A_y - A_x\right) = P_yA_y + \rho_yA_yv_y{}^2\,. \tag{A.9}$$

The first key reaction is used to convert the velocity to Mach number, and the second key relation is used to convert the density term to pressures and temperatures. These substitutions leads to

$$P_xA_x+\left(\frac{P_x}{RT_x}\right)A_x\left(M_x\sqrt{\gamma RT_x}\right)^2+P_s\left(A_y-A_x\right)=P_yA_y+\left(\frac{P_x}{RT_x}\right)A_y\left(M_y\sqrt{\gamma RT_y}\right)^2. \qquad \text{(A.10)}$$

Combine like terms.

$$P_xA_x+\gamma P_xA_xM_x^{\ 2}+P_s\left(A_y-A_x\right)=P_yA_y+\gamma P_yA_yM_y^{\ 2} \qquad \text{(A.11)}$$

Now, collect the PA terms on each side.

$$P_xA_x\left(1+\gamma M_x^{\ 2}\right)+P_s\left(A_y-A_x\right)=P_yA_y\left(1+\gamma M_y^{\ 2}\right) \qquad \text{(A.12)}$$

To get the ratio of inlet and exit pressure, several steps are needed. First, divide through by the $A_y\left(1+\gamma M_y\right)$ term. This leads to

$$P_y=\frac{P_xA_x\left(1+\gamma M_x^{\ 2}\right)}{A_y\left(1+\gamma M_y^{\ 2}\right)}+\frac{P_s\left(A_y-A_x\right)}{A_y\left(1+\gamma M_y^{\ 2}\right)}\ . \qquad \text{(A.13)}$$

Next, divide through by the P_x.

$$\frac{P_y}{P_x}=\frac{A_x\left(1+\gamma M_x^{\ 2}\right)}{A_y\left(1+\gamma M_y^{\ 2}\right)}+\frac{P_s\left(A_y-A_x\right)}{P_xA_y\left(1+\gamma M_y^{\ 2}\right)} \qquad \text{(A.14)}$$

The term with the side pressure can be simplified by consolidating the A_y, this leads to Equation 2.11.

$$\frac{P_y}{P_x}=\frac{A_x}{A_y}\frac{1+\gamma M_x^{\ 2}}{1+\gamma M_y^{\ 2}}+\frac{P_s}{P_x}\frac{1-\dfrac{A_x}{A_y}}{1+\gamma M_y^{\ 2}} \qquad \text{(2.11)}$$

Using the definition of r, the area ratios can be simplified.

$$\frac{P_y}{P_x} = r\frac{1+\gamma M_x{}^2}{1+\gamma M_y{}^2} + \frac{P_s}{P_x}\frac{1-r}{1+\gamma M_y{}^2} \tag{A.15}$$

Recall the definition of φ is $P_s = P_x + \varphi\left(P_y - P_x\right)$. Substitute this term into Equation A.15.

$$\frac{P_y}{P_x} = r\frac{1+\gamma M_x{}^2}{1+\gamma M_y{}^2} + \frac{P_x + \varphi\left(P_y - P_x\right)}{P_x}\frac{1-r}{1+\gamma M_y{}^2} \tag{A.16}$$

Equation 2.15 is the result of lengthy algebraic manipulation to isolate the pressure ratio. This was done in a computational program for accuracy. Equation 2.15 is reproduced below.

$$\frac{P_y}{P_x} = \frac{r(1+\gamma M_x{}^2)+(1-\phi)(1-r)}{(1+\gamma M_y{}^2)-\phi(1-r)} \tag{2.15}$$

Conservation of energy is given as

$$\frac{dE}{dt} = \sum \dot{W}_{in} + \sum \dot{Q}_{in} + \sum \dot{m}\left(h + \frac{v^2}{2} + gz\right)_{in} \quad . \tag{A.17}$$

Several simplifications can be made to Equation A.17. The system is steady-state, adiabatic, no significant height change, and has no work into the system. Since there is only one inlet or exit, the resulting form of the conservation of energy is

$$0 = \dot{m}_x\left(h + \frac{v^2}{2}\right)_y - \dot{m}_y\left(h + \frac{v^2}{2}\right)_y \; . \tag{A.18}$$

One of the consequences of this system having only one inlet and exit is that the mass flow is constant. This means that Equation A.18 can be simplified to

$$h_x + \frac{v_x{}^2}{2} = h_y + \frac{v_y{}^2}{2} \; . \tag{A.19}$$

This is a slight variation on Equation 2.7, as the specific heat addition has already been assumed to be zero. Next, the first key relation is used to change the velocity to Mach number. This results in

$$h_x + \frac{\left(M_x\sqrt{\gamma R T_x}\right)^2}{2} = h_y + \frac{\left(M_y\sqrt{\gamma R T_y}\right)^2}{2} \tag{A.20}$$

Simplify terms.

$$h_x + \frac{\gamma R T_x M_x^{\ 2}}{2} = h_y + \frac{\gamma R T_y M_y^{\ 2}}{2} \tag{A.21}$$

Since this model assumes an ideal gas, a change in enthalpy can be equated to temperature by the following, $\Delta h = c_p \Delta T$. To get the change in enthalpy, the exit enthalpy is subtracted to the other side. This is shown below in Equation A.22.

$$h_x - h_y + \frac{\gamma R T_x M_x^{\ 2}}{2} = \frac{\gamma R T_y M_y^{\ 2}}{2} \tag{A.22}$$

From Equation A.22 the ideal gas relation for enthalpy can be applied.

$$c_p\left(T_x - T_y\right) + \frac{\gamma R T_x M_x^{\ 2}}{2} = \frac{\gamma R T_y M_y^{\ 2}}{2} \tag{A.23}$$

There are two relations that allow for Equation A.23 to be simplified further. First is that γ is the ratio of specific heats, specifically, $\gamma = c_p / c_v$. Second, the specific gas constant, *R*, can be shown to be $R = c_p - c_v$. These two relations lead to a well-known relation; $c_p = (\gamma R)/(\gamma - 1)$. This can be substituted into Equation A.23.

$$\frac{\gamma R}{\gamma - 1}\left(T_x - T_y\right) + \frac{\gamma R T_x M_x^{\ 2}}{2} = \frac{\gamma R T_y M_y^{\ 2}}{2} \tag{A.24}$$

Multiply $\gamma - 1$ through Equation A.24.

$$\gamma R\left(T_x - T_y\right) + \frac{\gamma R T_x M_x^{\ 2}(\gamma - 1)}{2} = \frac{\gamma R T_y M_y^{\ 2}(\gamma - 1)}{2} \tag{A.25}$$

Distribute the γR term through the temperature difference.

$$\gamma RT_x - \gamma RT_y + \frac{\gamma RT_x M_x^{\,2}\left(\gamma - 1\right)}{2} = \frac{\gamma RT_y M_y^{\,2}\left(\gamma - 1\right)}{2} \tag{A.26}$$

Collect like terms.

$$\gamma RT_x\left(1 + \frac{M_x^{\,2}\left(\gamma - 1\right)}{2}\right) = \gamma RT_y\left(1 + \frac{M_y^{\,2}\left(\gamma - 1\right)}{2}\right) \tag{A.27}$$

Cancel the constants γ and R, and isolate the temperature ratio. These two steps lead to Equation 2.8, reproduced below.

$$\frac{T_y}{T_x} = \frac{1 + \left(\frac{\gamma - 1}{2}\right)M_x^{\,2}}{1 + \left(\frac{\gamma - 1}{2}\right)M_y^{\,2}} \tag{2.8}$$

Appendix B

Appendix B shows the commented MATLAB code used to create Figures **2.3** through **2.11**.

```
%This code used Equation 2.8, 2.14, 2.18, 2.19, and 2.20 to calculate the
%values for the exit Mach number, pressure ratio, and temperature ratio
%over various area ratios, r, and inlet Mach numbers, Mx. The results are
%plotted in three separate plots.

%Initialization code
clc
clear variables
close all

%Constants needed for the calculations. Phi is set to one half because the
%pressure is assumed to change linearly. Gamma was chosen because air is
%the theoretical working fluid.
gamma = 1.4;
phi = 0.5;
cp = 1.005; %kJ/(kg-K)
R = .2869; %kJ/(kg-K)

%% ------------------------Area Ratio Dependence------------------
%This determines the range of inlet m=Mach number and different r values that
%are being investigated.
Mx = 1:0.01:5;
r = [0.5,0.75,1.0,2.0];

%This loop increments over each r value and calculates the exit Mach number
```

```
% and the pressure and temperature ratios for all of the inlet Mach numbers
for i = 1:length(r)

    %The value for B at each r value.
    B = phi*(1-r(i));

    %The calculation for A at each r value for every inlet Mach number
    ANum = Mx.^2.*r(i)^2.*(2+Mx.^2.*(gamma-1));
    ADem = ((1+gamma.*Mx.^2).*r(i)+(1-phi)*(1-r(i))).^2;
    Ar = ANum./ADem;

    %The calculation for the exit Mach number based on A and B found prior
    MyNum = Ar.*gamma.*(1-B)-1+sqrt(1-2*gamma.*Ar.*(1-B)+Ar.*(gamma-1).*((1-
B).^2));
    MyDem = gamma-1-Ar.*gamma^2;
    Myr(:,i) = real(sqrt(MyNum./MyDem));

    %The calculations for the pressure ratio
    PyPxNum = r(i)*(1+gamma.*Mx.^2)+(1-phi)*(1-r(i));
    PyPxDem = (1+gamma.*Myr(:,i).^2)-phi*(1-r(i));
    PyPxr(:,i) = PyPxNum./PyPxDem';

    %The calculation for the temperature ratio
    TyTxNum = 1+Mx.^2.*((gamma-1)/2);
    TyTxDem = 1+Myr(:,i).^2.*((gamma-1)/2);
    TyTxr(:,i) = TyTxNum./TyTxDem';

    Sgenr(:,i) = cp*log(TyTxr(:,i))-R*log(PyPxr(:,i));
```

```
end

%This section calculates the exit Mach number based on the Anderson
%normal shock relations
MySNum = 1+((gamma-1)/2).*Mx.^2;
MySDem = gamma.*Mx.^2-(gamma-1)/2;
MyS = sqrt(MySNum./MySDem);

%This section calculates the pressure ratio based on the Anderson
%normal shock relations
PyPxS = 1+(2*gamma)/(gamma+1).*(Mx.^2-1);

%This section calculates the temperature based on the Anderson
%normal shock relations
TyTxS = PyPxS.*((2+(gamma-1).*Mx.^2)./((gamma+1).*Mx.^2));

%Plot of the exit Mach number versus the inlet Mach number. The r values
%are 0.5, 0.75, 0.9, 1.0. Where 1.0 is the shockwave solution.
plot(Mx,Myr(:,1),':')
hold on
plot(Mx,Myr(:,2),'-.')
plot(Mx,Myr(:,3),'--')
plot(Mx,Myr(:,4))
%Formating for the plot.
xlabel('Inlet Mach Number, Mx')
ylabel('Exit Mach Number, My')
legend('r = 0.50','r = 0.75','r = 0.90','r = 1.0')
```

```
%Plot of the pressure ratio versus the inlet Mach number. The r values
%are 0.5, 0.75, 0.9, 1.0. Where 1.0 is the shockwave solution.
figure
plot(Mx,PyPxr(:,1),':')
hold on
plot(Mx,PyPxr(:,2),'-.')
plot(Mx,PyPxr(:,3),'--')
plot(Mx,PyPxr(:,4))
%Formating for the plot.
xlabel('Inlet Mach Number, Mx')
ylabel('Pressure Ratio, Py/Px')
legend('r = 0.50','r = 0.75','r = 0.90','r = 1.0')

%Plot of the temperature at versus the inlet Mach number. The r values
%are 0.5, 0.75, 0.9, 1.0. Where 1.0 is the shockwave solution.
figure
plot(Mx,TyTxr(:,1),':')
hold on
plot(Mx,TyTxr(:,2),'-.')
plot(Mx,TyTxr(:,3),'--')
plot(Mx,TyTxr(:,4))
%Formating for the plot.
xlabel('Inlet Mach Number, Mx')
ylabel('Temperature Ratio, Ty/Tx')
legend('r = 0.50','r = 0.75','r = 0.90','r = 1.0')

%Plot of the entropy generation at versus the inlet Mach number. The r
%values are 0.5, 0.75, 0.9, 1.0. Where 1.0 is the shockwave solution.
```

```
figure
plot(Mx,Sgenr(:,1),':')
hold on
plot(Mx,Sgenr(:,2),'-.')
plot(Mx,Sgenr(:,3),'--')
plot(Mx,Sgenr(:,4))
%Formating for the plot.
xlabel('Inlet Mach Number, Mx')
ylabel('Entropy generation, Sgen (kJ/kg/K')
legend('r = 0.50','r = 0.75','r = 0.90','r = 1.0')

%% ---------------------Side Pressure Dependence------------------

%This determines the range of inlet Mach number and different Phi values
%that are being investigated.
r = 0.9;
Phi = [-0.5,0.0,1.0,2.0];

%This loop increments over each r value and calculates the exit Mach number
% and the pressure and temperature ratios for all of the inlet Mach numbers
for i = 1:length(Phi)

    %The value for B at each r value.
    B = Phi(i)*(1-r);

    %The calculation for A at each r value for every inlet Mach number
    ANum = Mx.^2.*r^2.*(2+Mx.^2.*(gamma-1));
    ADem = ((1+gamma.*Mx.^2).*r+(1-Phi(i))*(1-r)).^2;
```

```
    Ap = ANum./ADem;

    %The calculation for the exit Mach number based on A and B found prior
    MyNum = Ap.*gamma.*(1-B)-1+sqrt(1-2*gamma.*Ap.*(1-B)+Ap.*(gamma-1).*((1-
B).^2));
    MyDem = gamma-1-Ap.*gamma^2;
    Myp(:,i) = real(sqrt(MyNum./MyDem));

    %The calculations for the pressure ratio
    PyPxNum = r*(1+gamma.*Mx.^2)+(1-Phi(i))*(1-r);
    PyPxDem = (1+gamma.*Myp(:,i).^2)-Phi(i)*(1-r);
    PyPxp(:,i) = PyPxNum./PyPxDem';

    %The calcuation for the temperature ratio
    TyTxNum = 1+Mx.^2.*((gamma-1)/2);
    TyTxDem = 1+Myp(:,i).^2.*((gamma-1)/2);
    TyTxp(:,i) = TyTxNum./TyTxDem';

    Sgenp(:,i) = cp*log(TyTxp(:,i))-R*log(PyPxp(:,i));

    PsPx(:,i) = (1+Phi(i)*(PyPxp(:,i)-1));
end

%Concatenating all of the data that is to be saved to a file
%"PerlimVerificationData.mat"
```

```
DataMatrix =
[Mx',Myr,Myp,MyS',PyPxr,PyPxp,PyPxS',TyTxp,TyTxp,TyTxS',Sgenr,Sgenp];
save('PrelimVerificationData.mat','DataMatrix');



%Plot of the exit Mach number versus the inlet Mach number. The Phi values
%are -0.5, 0.0, 1.0, 2.0 as well as the shockwave solution.
figure
plot(Mx,MyS,'-r')
hold on
plot(Mx,Myp(:,1),':')
plot(Mx,Myp(:,2),'-.')
plot(Mx,Myp(:,3),'--')
plot(Mx,Myp(:,4))
%Formating for the plot.
xlabel('Inlet Mach Number, Mx')
ylabel('Exit Mach Number, My')
legend('Shock wave','Phi = -0.5','Phi = 0','Phi = 1.0','Phi = 2.0')

%Plot of the Pressure ratio versus the inlet Mach number. The Phi values
%are -0.5, 0.0, 1.0, 2.0 as well as the shockwave solution.
figure
plot(Mx,PyPxS,'-r')
hold on
plot(Mx,PyPxp(:,1),':')
plot(Mx,PyPxp(:,2),'-.')
plot(Mx,PyPxp(:,3),'--')
```

```
plot(Mx,PyPxp(:,4))
%Formating for the plot.
xlabel('Inlet Mach Number, Mx')
ylabel('Pressure Ratio, Py/Px')
legend('Shock wave','Phi = -0.5','Phi = 0','Phi = 1.0','Phi = 2.0')

%Plot of the Temperature ratio versus the inlet Mach number. The Phi values
%are -0.5, 0.0, 1.0, 2.0 as well as the shockwave solution.
figure
plot(Mx,TyTxS,'-r')
hold on
plot(Mx,TyTxp(:,1),':')
plot(Mx,TyTxp(:,2),'-.')
plot(Mx,TyTxp(:,3),'--')
plot(Mx,TyTxp(:,4))
%Formating for the plot.
xlabel('Inlet Mach Number, Mx')
ylabel('Temperature Ratio, Ty/Tx')
legend('Shock wave','Phi = -0.5','Phi = 0','Phi = 1.0','Phi = 2.0')

%Plot of the entropy generation versus the inlet Mach number. The Phi values
%are -0.5, 0.0, 1.0, 2.0 as well as the shockwave solution.
figure
plot(Mx,Sgenr(:,1),'-r')
hold on
plot(Mx,Sgenp(:,1),':')
plot(Mx,Sgenp(:,2),'-.')
plot(Mx,Sgenp(:,3),'--')
```

```
plot(Mx,Sgenp(:,4))
%Formatting for the plot.
xlabel('Inlet Mach Number, Mx')
ylabel('Entropy generation, Sgen (kJ/kg/K')
legend('Shock wave','Phi = -0.5','Phi = 0','Phi = 1.0','Phi = 2.0')

figure
plot(Mx,PsPx(:,1),':')
hold on
plot(Mx,PsPx(:,2),'-.')
plot(Mx,PsPx(:,3),'--')
plot(Mx,PsPx(:,4))
%Formatting for the plot.
xlabel('Inlet Mach Number, Mx')
ylabel('Side Pressure Ratio, Py/Px')
legend('Phi = -0.5','Phi = 0','Phi = 1.0','Phi = 2.0')

%% Saving Data to Excel
%DataMatrix =
[Mx',Myr,Myp,MyS',PyPxr,PyPxp,PyPxS',TyTxp,TyTxp,TyTxS',Sgenr,Sgenp];

T1 = table(Mx',MyS',Myr);
Filename = 'PreliminaryData.xlsx';
writetable(T1,Filename,'Sheet',1)

T2 = table(Mx',PyPxS',PyPxr);
Filename = 'PreliminaryData.xlsx';
writetable(T2,Filename,'Sheet',2)
```

```
T3 = table(Mx',TyTxS',TyTxr);
Filename = 'PreliminaryData.xlsx';
writetable(T3,Filename,'Sheet',3)

T4 = table(Mx',Sgenr);
Filename = 'PreliminaryData.xlsx';
writetable(T4,Filename,'Sheet',4)

T5 = table(Mx',MyS',Myp);
Filename = 'PreliminaryData.xlsx';
writetable(T5,Filename,'Sheet',5)

T6 = table(Mx',PyPxS',PyPxp);
Filename = 'PreliminaryData.xlsx';
writetable(T6,Filename,'Sheet',6)


T7 = table(Mx',TyTxS',TyTxp);
Filename = 'PreliminaryData.xlsx';
writetable(T7,Filename,'Sheet',7)

T8 = table(Mx',Sgenp);
Filename = 'PreliminaryData.xlsx';
writetable(T8,Filename,'Sheet',8)
```

Tables B.1 through B.9 contains the raw data produced from the MATLAB code shown prior in Appendix B.

Table B.1: Exit Mach Number for Varying Inlet Mach Numbers and Area Ratios

M_x	Shock Wave	R = 0.5	R = 0.75	R = 0.9	R = 1.0
1.0000	1.0000	0.3408	0.5225	0.6866	1.0000
1.1000	0.9118	0.3491	0.5273	0.6851	0.9118
1.2000	0.8422	0.3541	0.5256	0.6715	0.8422
1.3000	0.7860	0.3564	0.5194	0.6518	0.7860
1.4000	0.7397	0.3567	0.5106	0.6301	0.7397
1.5000	0.7011	0.3556	0.5005	0.6086	0.7011
1.6000	0.6684	0.3536	0.4899	0.5883	0.6684
1.7000	0.6405	0.3509	0.4794	0.5696	0.6405
1.8000	0.6165	0.3478	0.4692	0.5527	0.6165
1.9000	0.5956	0.3444	0.4596	0.5374	0.5956
2.0000	0.5774	0.3410	0.4507	0.5236	0.5774
2.1000	0.5613	0.3377	0.4423	0.5111	0.5613
2.2000	0.5471	0.3343	0.4346	0.4999	0.5471
2.3000	0.5344	0.3311	0.4275	0.4898	0.5344
2.4000	0.5231	0.3280	0.4210	0.4806	0.5231
2.5000	0.5130	0.3251	0.4150	0.4723	0.5130
2.6000	0.5039	0.3223	0.4094	0.4648	0.5039
2.7000	0.4956	0.3197	0.4043	0.4579	0.4956
2.8000	0.4882	0.3172	0.3996	0.4517	0.4882
2.9000	0.4814	0.3149	0.3953	0.4460	0.4814
3.0000	0.4752	0.3127	0.3913	0.4407	0.4752

Table B.2: Pressure Ratios for Varying Inlet Mach Numbers and Area Ratios

M_x	Shock Wave	R = 0.5	R = 0.75	R = 0.9	R = 1.0
1.0000	1.0000	1.5889	1.5312	1.3727	1.0000
1.1000	1.2450	1.7347	1.6970	1.5398	1.2450
1.2000	1.5133	1.8995	1.8919	1.7483	1.5133
1.3000	1.8050	2.0834	2.1151	1.9935	1.8050
1.4000	2.1200	2.2863	2.3653	2.2710	2.1200
1.5000	2.4583	2.5079	2.6414	2.5775	2.4583
1.6000	2.8200	2.7481	2.9422	2.9108	2.8200
1.7000	3.2050	3.0065	3.2668	3.2696	3.2050
1.8000	3.6133	3.2829	3.6146	3.6529	3.6133
1.9000	4.0450	3.5772	3.9849	4.0602	4.0450
2.0000	4.5000	3.8890	4.3775	4.4910	4.5000
2.1000	4.9783	4.2183	4.7919	4.9451	4.9783
2.2000	5.4800	4.5649	5.2279	5.4223	5.4800
2.3000	6.0050	4.9286	5.6854	5.9224	6.0050
2.4000	6.5533	5.3095	6.1641	6.4453	6.5533
2.5000	7.1250	5.7073	6.6640	6.9909	7.1250
2.6000	7.7200	6.1220	7.1850	7.5592	7.7200
2.7000	8.3383	6.5536	7.7270	8.1500	8.3383
2.8000	8.9800	7.0020	8.2899	8.7635	8.9800
2.9000	9.6450	7.4671	8.8736	9.3994	9.6450
3.0000	10.3333	7.9489	9.4782	10.0579	10.3333

Table B.3: Temperature Ratios for Varying Inlet Mach Numbers and Area Ratios

M_x	Shock Wave	R = 0.5	R = 0.75	R = 0.9	R = 1.0
1.0000	1.0000	1.1728	1.1379	1.0966	1.0000
1.1000	1.0649	1.2124	1.1766	1.1354	1.0649
1.2000	1.1280	1.2565	1.2206	1.1815	1.1280
1.3000	1.1909	1.3049	1.2695	1.2332	1.1909
1.4000	1.2547	1.3575	1.3230	1.2896	1.2547
1.5000	1.3202	1.4142	1.3808	1.3500	1.3202
1.6000	1.3880	1.4751	1.4427	1.4141	1.3880
1.7000	1.4583	1.5401	1.5087	1.4818	1.4583
1.8000	1.5316	1.6091	1.5785	1.5531	1.5316
1.9000	1.6079	1.6821	1.6522	1.6280	1.6079
2.0000	1.6875	1.7591	1.7297	1.7064	1.6875
2.1000	1.7705	1.8400	1.8111	1.7885	1.7705
2.2000	1.8569	1.9250	1.8964	1.8743	1.8569
2.3000	1.9468	2.0138	1.9854	1.9638	1.9468
2.4000	2.0403	2.1067	2.0783	2.0570	2.0403
2.5000	2.1375	2.2034	2.1751	2.1539	2.1375
2.6000	2.2383	2.3041	2.2757	2.2546	2.2383
2.7000	2.3429	2.4088	2.3802	2.3591	2.3429
2.8000	2.4512	2.5173	2.4885	2.4673	2.4512
2.9000	2.5632	2.6298	2.6007	2.5794	2.5632
3.0000	2.6790	2.7463	2.7168	2.6953	2.6790

Table B.4: Entropy Generation for Varying Inlet Mach Numbers and Area Ratios

M_x	R = 0.5	R = 0.75	R = 0.9	R = 1.0
1.0000	0.0273	0.0076	0.0018	0.0000
1.1000	0.0356	0.0117	0.0038	0.0004
1.2000	0.0454	0.0174	0.0073	0.0022
1.3000	0.0568	0.0249	0.0128	0.0061
1.4000	0.0699	0.0343	0.0203	0.0124
1.5000	0.0845	0.0456	0.0300	0.0211
1.6000	0.1007	0.0588	0.0417	0.0320
1.7000	0.1182	0.0736	0.0554	0.0450
1.8000	0.1370	0.0901	0.0708	0.0599
1.9000	0.1570	0.1080	0.0878	0.0764
2.0000	0.1780	0.1271	0.1061	0.0943
2.1000	0.1999	0.1474	0.1257	0.1136
2.2000	0.2226	0.1686	0.1464	0.1339
2.3000	0.2459	0.1907	0.1679	0.1552
2.4000	0.2699	0.2134	0.1902	0.1773
2.5000	0.2943	0.2368	0.2132	0.2001
2.6000	0.3190	0.2606	0.2367	0.2234
2.7000	0.3441	0.2849	0.2606	0.2472
2.8000	0.3695	0.3094	0.2849	0.2713
2.9000	0.3949	0.3342	0.3095	0.2957
3.0000	0.4205	0.3592	0.3342	0.3204

Table B.5: Exit Mach Number for Varying Inlet Mach Numbers and Phi Values

M_x	Shock Wave	Phi = -0.5	Phi = 0.0	Phi = 1.0	Phi = 2.0
1.0000	1.0000	0.7252	0.7071	0.6635	0.6097
1.1000	0.9118	0.7364	0.7119	0.6562	0.5926
1.2000	0.8422	0.7323	0.7027	0.6388	0.5702
1.3000	0.7860	0.7182	0.6855	0.6173	0.5470
1.4000	0.7397	0.6992	0.6648	0.5950	0.5248
1.5000	0.7011	0.6785	0.6435	0.5737	0.5044
1.6000	0.6684	0.6579	0.6230	0.5539	0.4860
1.7000	0.6405	0.6384	0.6038	0.5358	0.4695
1.8000	0.6165	0.6203	0.5863	0.5196	0.4548
1.9000	0.5956	0.6038	0.5703	0.5050	0.4417
2.0000	0.5774	0.5887	0.5559	0.4918	0.4300
2.1000	0.5613	0.5751	0.5428	0.4800	0.4195
2.2000	0.5471	0.5627	0.5310	0.4694	0.4101
2.3000	0.5344	0.5515	0.5203	0.4598	0.4016
2.4000	0.5231	0.5413	0.5107	0.4512	0.3940
2.5000	0.5130	0.5321	0.5019	0.4434	0.3871
2.6000	0.5039	0.5237	0.4939	0.4363	0.3808
2.7000	0.4956	0.5160	0.4867	0.4298	0.3752
2.8000	0.4882	0.5090	0.4800	0.4239	0.3700
2.9000	0.4814	0.5026	0.4740	0.4185	0.3653
3.0000	0.4752	0.4967	0.4684	0.4136	0.3609

Table B.6: Pressure Ratios for Varying Inlet Mach Numbers and Phi Values

M_x	Shock Wave	Phi = -0.5	Phi = 0.0	Phi = 1.0	Phi = 2.0
1.0000	1.0000	1.2932	1.3295	1.4244	1.5602
1.1000	1.2450	1.4231	1.4768	1.6134	1.7998
1.2000	1.5133	1.5907	1.6641	1.8448	2.0828
1.3000	1.8050	1.7941	1.8877	2.1133	2.4034
1.4000	2.1200	2.0293	2.1433	2.4143	2.7579
1.5000	2.4583	2.2928	2.4275	2.7449	3.1441
1.6000	2.8200	2.5819	2.7379	3.1032	3.5605
1.7000	3.2050	2.8948	3.0728	3.4881	4.0063
1.8000	3.6133	3.2305	3.4312	3.8988	4.4810
1.9000	4.0450	3.5880	3.8125	4.3347	4.9842
2.0000	4.5000	3.9668	4.2162	4.7955	5.5156
2.1000	4.9783	4.3666	4.6418	5.2810	6.0750
2.2000	5.4800	4.7870	5.0893	5.7910	6.6624
2.3000	6.0050	5.2279	5.5584	6.3254	7.2775
2.4000	6.5533	5.6892	6.0490	6.8840	7.9204
2.5000	7.1250	6.1706	6.5610	7.4667	8.5910
2.6000	7.7200	6.6722	7.0944	8.0737	9.2892
2.7000	8.3383	7.1939	7.6490	8.7047	10.0150
2.8000	8.9800	7.7356	8.2248	9.3597	10.7684
2.9000	9.6450	8.2973	8.8219	10.0388	11.5493
3.0000	10.3333	8.8789	9.4401	10.7419	12.3578

Table B.7: Temperature Ratios for Varying Inlet Mach Numbers and Phi Values

M_x	Shock Wave	Phi = -0.5	Phi = 0.0	Phi = 1.0	Phi = 2.0
1.0000	1.0000	1.0858	1.0909	1.1029	1.1170
1.1000	1.0649	1.1205	1.1277	1.1435	1.1605
1.2000	1.1280	1.1633	1.1722	1.1908	1.2094
1.3000	1.1909	1.2129	1.2231	1.2432	1.2625
1.4000	1.2547	1.2680	1.2789	1.2999	1.3193
1.5000	1.3202	1.3278	1.3391	1.3605	1.3798
1.6000	1.3880	1.3915	1.4031	1.4246	1.4438
1.7000	1.4583	1.4591	1.4707	1.4923	1.5114
1.8000	1.5316	1.5302	1.5420	1.5636	1.5825
1.9000	1.6079	1.6050	1.6168	1.6384	1.6573
2.0000	1.6875	1.6833	1.6952	1.7169	1.7358
2.1000	1.7705	1.7652	1.7773	1.7991	1.8180
2.2000	1.8569	1.8508	1.8629	1.8849	1.9040
2.3000	1.9468	1.9400	1.9523	1.9745	1.9937
2.4000	2.0403	2.0329	2.0453	2.0678	2.0872
2.5000	2.1375	2.1294	2.1421	2.1649	2.1845
2.6000	2.2383	2.2297	2.2426	2.2658	2.2857
2.7000	2.3429	2.3337	2.3468	2.3704	2.3907
2.8000	2.4512	2.4415	2.4549	2.4789	2.4996
2.9000	2.5632	2.5530	2.5667	2.5912	2.6123
3.0000	2.6790	2.6683	2.6823	2.7074	2.7289

Table B.8: Entropy Generated for Varying Inlet Mach Numbers and Phi Values

M_x	Phi = -0.5	Phi = 0.0	Phi = 1.0	Phi = 2.0
1.0000	0.0090	0.0058	-0.0031	-0.0164
1.1000	0.0131	0.0089	-0.0024	-0.0190
1.2000	0.0188	0.0136	-0.0002	-0.0195
1.3000	0.0263	0.0201	0.0041	-0.0174
1.4000	0.0356	0.0285	0.0108	-0.0125
1.5000	0.0469	0.0390	0.0197	-0.0051
1.6000	0.0599	0.0514	0.0308	0.0048
1.7000	0.0747	0.0656	0.0439	0.0169
1.8000	0.0911	0.0815	0.0588	0.0310
1.9000	0.1089	0.0989	0.0754	0.0469
2.0000	0.1280	0.1176	0.0935	0.0643
2.1000	0.1482	0.1375	0.1128	0.0831
2.2000	0.1694	0.1584	0.1332	0.1031
2.3000	0.1915	0.1802	0.1545	0.1240
2.4000	0.2142	0.2027	0.1766	0.1458
2.5000	0.2375	0.2259	0.1994	0.1683
2.6000	0.2614	0.2495	0.2228	0.1913
2.7000	0.2856	0.2736	0.2466	0.2149
2.8000	0.3101	0.2980	0.2707	0.2388
2.9000	0.3349	0.3227	0.2952	0.2631
3.0000	0.3599	0.3475	0.3198	0.2876

Table B.9: Side Wall Pressure Ratio for Varying Inlet Mach Numbers and Phi Values

M_x	Phi = -0.5	Phi = 0.0	Phi = 1.0	Phi = 2.0
1.0000	0.8534	1.0000	1.4244	2.1204
1.1000	0.7885	1.0000	1.6134	2.5997
1.2000	0.7046	1.0000	1.8448	3.1656
1.3000	0.6030	1.0000	2.1133	3.8068
1.4000	0.4854	1.0000	2.4143	4.5158
1.5000	0.3536	1.0000	2.7449	5.2881
1.6000	0.2091	1.0000	3.1032	6.1210
1.7000	0.0526	1.0000	3.4881	7.0127
1.8000	-0.1152	1.0000	3.8988	7.9621
1.9000	-0.2940	1.0000	4.3347	8.9684
2.0000	-0.4834	1.0000	4.7955	10.0312
2.1000	-0.6833	1.0000	5.2810	11.1500
2.2000	-0.8935	1.0000	5.7910	12.3247
2.3000	-1.1140	1.0000	6.3254	13.5550
2.4000	-1.3446	1.0000	6.8840	14.8408
2.5000	-1.5853	1.0000	7.4667	16.1819
2.6000	-1.8361	1.0000	8.0737	17.5784
2.7000	-2.0970	1.0000	8.7047	19.0300
2.8000	-2.3678	1.0000	9.3597	20.5368
2.9000	-2.6486	1.0000	10.0388	22.0987
3.0000	-2.9395	1.0000	10.7419	23.7157

Appendix C

Appendix C shows all the code and data to create the figures 3.2 and 3.3. The function PhiCalculator.m is the function that uses the methods described in 3.2.

```
function [InletMach,Phi]=PhiCalculator(r,Pr)
%[InletMach,Phi]=PhiCalculator(r,Pr)
%This function analizes Conservation of Linear Momentum (CoLM) across a
%small section of a nozzle. The nozzle section is assumed to be a straight
%line, such that there is no throat in the section.
%
% ------------------------------Inputs-------------------------------------
%
%r is the area ratio of the nozzle section that is being analyized. R can
%only be possitive and less than 1.0. A r value of 1.0 means that both ends
%of the nozzle are of equal area. R values of 0 or negative is
%non-physical.
%
%Pr is the total pressure ratio of the nozzle ends, where the total
%pressure of the outlet is on top of the inlet. This means that Pr is
%required to be less than 1 for all real flows where entropy is generated.
%A Pr of 1 means that the flow is isentropic.
%
% -----------------------------Outputs-------------------------------------
%
%InletMach is the Mach number of the inlet to the nozzle section. This is
%used for the graphing x-axis, but is not critical a critical output.
%Change this to look a smaller section oof nozzle speeds
```

```
%
%Phi is the parameter for the side wall pressure that is defined as:
%                    P_side = P_in+Phi*(P_out-P_in)
%This means that if Phi is between 1 and zero, the pressure at the side
%of the nozzle is between the inlet and exit pressure.

%% -----------------------Constants and Variables--------------------------

%These can be changed to fit the system, but are currently at simple
%values with no meaningful demensionality.
A1 = 1.0; %DISTANCE SQUARED
P0 = 100; %PRESSURE
gamma = 1.4; %Unitless

%Initialization and Definitons for the code.
A2 = A1/r; %DISTANCE SQUARED
k = 1; %Unitsless indexing variable

%% --------------------Exit Mach Number Calculation------------------------

%This for loop will vary the inlet Mach number of the nozzle from a stated
%initial speed to a final speed with a given step size.
for MachRange = 0.1:0.1:2
    M(k,1) = MachRange;

    %This section creates a symbolic equation that will solve for the exit
    %Mach number given the stated constants and inlet Mach number passed in
    %by the for loop.
```

```
    syms Me Mi real
        MachEqn = (r/Pr)*M(k,1)/(1+((gamma-1)/2)*M(k,1)^2)^((0.5*(gamma+1))/(gamma-1))==...
            Me/(1+(gamma-1)/2*Me^2)^((0.5*(gamma+1))/(gamma-1));
    ExitMachInter = vpasolve(MachEqn,Me,[0 Inf]);
     ExitMach = double(ExitMachInter);

    %Due to the fact that the symbolic equation has two real solutions at
    %every inlet Mach number, this code segment picks out the correct
    %value. We are assuming that there is no shock in the nozzle, so that
    %the realizable soution is in the same flow regime (super sonic or
    %subsonic).
    if M(k,1) < 1
        M(k,2) = min(ExitMach);
    elseif M(k,1) >= 1
        M(k,2) = max(ExitMach);
    end

    %Increment the indexing number.
    k = k+1;
end

%% -------------------------CoLM Analysis----------------------------------

%Static pressure of both the exit and the inlet of the nozzle. These
%relations are based upon the isentropic realtions from Anderson. Note: Pr
%is used to realate the total pressure at the exit to the total pressure at
%the inlet.
```

```
P1 = P0./(1+((gamma-1)/2)*M(:,1).^2).^(gamma/(gamma-1)); %kpa
P2 = (P0*Pr)./(1+((gamma-1)/2)*M(:,2).^2).^(gamma/(gamma-1)); %kpa

%Calcuations for the inlet and exit MDot*V in momentum. The Mach number
%relations are from Anderson, and some algebra to get the equation into
%this form.
MDotVin = gamma*P0*A1*M(:,1).^2./(1+((gamma-1)/2)*M(:,1).^2).^(gamma/(gamma-
1));
MDotVout = gamma*(P0*Pr)*A2.*M(:,2).^2./(1+((gamma-
1)/2).*M(:,2).^2).^(gamma/(gamma-1));

%Defines The momentum added by the Static Pressure at the inlet and exit.
PAin = P1*A1;
PAout = P2*A2;

%Solution for the Side Pressure of the nozzle from CoLM
Ps = (PAout-PAin+MDotVout-MDotVin)./(A2-A1);

%This sets the exit parameters for the function.
InletMach = M(:,1);
Phi = (Ps(:)-P1)./(P2-P1);
```

The main script PhiGraphs.m is the overall function that creates the figures and data.

```
%% PhiGraphs
```

```
%This Code calls the PhiCalculator function for various area ratios, and
%total pressure loss. The first case shows how Phi change with differing r
%values. The Third case shows how phi changes when entropy is generated.
%This entropy generation is shown as a loss of total pressure.
%
% -----------------------Function Passthrough Inputs-----------------------
%
%r is the area ratio of the nozzle section that is being analyized. R can
%only be possitive and less than 1.0. A r value of 1.0 means that both ends
%of the nozzle are of equal area. R values of 0 or negative are
%non-physical.
%
%Pr is the total pressure ratio of the nozzle ends, where the total
%pressure of the outlet is over of the inlet. This means that Pr is
%required to be less than 1 for all real flows where entropy is generated.
%A Pr of 1 means that the flow is isentropic.
%
% ----------------------Function Passthrough Outputs-----------------------
%
%Mx is the Mach number of the inlet to the nozzle section. This is
%used for the graphing x-axis, but is not critical a critical output.
%Change this to look a smaller section oof nozzle speeds
%
%Phi is the parameter for the side wall pressure that is defined as:
%                   P_side = P_in+Phi*(P_out-P_in)
%This means that if Phi is between 1 and zero, the pressure at the side
%of the nozzle is between the inlet and exit pressure.

%% Initialization Code
```

```
%Clearing prior info.

clc

clear variables

close all


%% ----------------------Isentropic Phi Values----------------------------


%Inputs for this isentropic cases. This will show the change in phi values

%as the area ratio, r, changes.

r = [0.95, 0.9, 0.75, 0.5];

Pr = 1.0;


%Calls the PhiCalculator function several times so that it can generate and

%plot all of the varying r cases.

for i=1:4

    [Mx(i,:),PhiIce(i,:)]=PhiCalculator(r(i),Pr);

end


%The plot commands so that all four lines are ploted on the same figure.

figure(2)

plot(Mx(1,:),PhiIce(1,:))

hold on

plot(Mx(2,:),PhiIce(2,:),'--')

plot(Mx(3,:),PhiIce(3,:),'-.')

plot(Mx(4,:),PhiIce(4,:),':')

hold off


%Formatting for the Second figure
```

```
axis([0 2.0 0 1.0]);
xlabel('Inlet Mach Number')
ylabel('Isentropic \Phi Value')
%title('Isentropic \Phi versus Area Ratio and Inlet Mach #') %NEEDS TO BE REMOVED
FOR IEEE FORMAT
legend('r = 0.95', 'r = 0.9', 'r = 0.75', 'r = 0.5')

%% --------------------Entropy Generated Phi Values------------------------

%Inputs for this isentropic cases. This will show how the Phi value will
%change as more entropy is generated in the system
r = 0.9;
Pr = [1.0,0.9999,0.9995,0.999];

%Calls the PhiCalculator function several times so that it can generate and
%plot all of different entropy generations
for i=1:4
    [Mx(i,:),PhiSGen(i,:)]=PhiCalculator(r,Pr(i));
end

%This is a varible to show the amount of entropy being generated with each
%iteration
Sgen = -286.9*log(Pr); %J/kgK

%The plot commands so that all of the different entropic cases are in the
%same graph.
figure(3)
plot(Mx(1,:),PhiSGen(1,:))
```

```
hold on
plot(Mx(2,:),PhiSGen(2,:),'--')
plot(Mx(3,:),PhiSGen(3,:),'-.')
plot(Mx(4,:),PhiSGen(4,:),':')
hold off

%Formatting for the Third Figure.
axis([0 2.0 0 1.0]);
xlabel('Inlet Mach Number')
ylabel('\Phi Value')
%title('\Phi versus Total Pressure Loss and Inlet Mach # at r = 0.9') %NEEDS TO
BE REMOVED FOR IEEE
legend('Isentropic','0.01% Total Pressure Loss', '0.05% Total Pressure
Loss','0.1% Total Pressure Loss')
```

The tables for the data used in graphs 3.2 and 3.3.

Table C.1: The Phi Values for Various Area Ratios and Inlet Mach Numbers for an Isentropic Flow

Inlet Mach Number	Phi, r = 0.95	Phi, r = 0.90	Phi, r = 0.75	Phi, r = 0.50
0.1000	0.5129	0.5266	0.5720	0.6675
0.2000	0.5133	0.5273	0.5737	0.6701
0.3000	0.5141	0.5288	0.5768	0.6745
0.4000	0.5154	0.5312	0.5817	0.6812
0.5000	0.5177	0.5353	0.5892	0.6904
0.6000	0.5217	0.5421	0.6004	0.7027
0.7000	0.5296	0.5541	0.6170	0.7185
0.8000	0.5463	0.5760	0.6412	0.7384
0.9000	0.5854	0.6163	0.6757	0.7627
1.0000	0.6752	0.6848	0.7162	0.7795
1.1000	0.5868	0.6162	0.6715	0.7519
1.2000	0.5503	0.5797	0.6412	0.7305
1.3000	0.5340	0.5596	0.6205	0.7140
1.4000	0.5258	0.5480	0.6062	0.7010
1.5000	0.5212	0.5409	0.5960	0.6908
1.6000	0.5184	0.5362	0.5886	0.6827
1.7000	0.5166	0.5331	0.5831	0.6761
1.8000	0.5154	0.5308	0.5789	0.6708
1.9000	0.5144	0.5292	0.5757	0.6664
2.0000	0.5138	0.5279	0.5732	0.6628

Table C.2: Phi Values for Various Total Pressure Ratios at an Area Ratio of 0.9

Inlet Mach Number	Phi, Pr = 1.0	Phi, Pr = 0.99	Phi, Pr = 0.95	Phi, Pr = 0.90
0.1000	0.5266	-0.2378	-5.1494	-28.3127
0.2000	0.5273	0.3515	-0.4261	-1.6047
0.3000	0.5288	0.4545	0.1442	-0.2768
0.4000	0.5312	0.4920	0.3311	0.1206
0.5000	0.5353	0.5121	0.4179	0.2967
0.6000	0.5421	0.5276	0.4688	0.3937
0.7000	0.5541	0.5447	0.5066	0.4582
0.8000	0.5760	0.5697	0.5444	0.5125
0.9000	0.6163	0.6121	0.5949	0.5732
1.0000	0.6848	0.6865	0.6933	0.7018
1.1000	0.6162	0.6184	0.6272	0.6382
1.2000	0.5797	0.5823	0.5930	0.6064
1.3000	0.5596	0.5627	0.5752	0.5908
1.4000	0.5480	0.5515	0.5655	0.5831
1.5000	0.5409	0.5447	0.5600	0.5793
1.6000	0.5362	0.5403	0.5568	0.5775
1.7000	0.5331	0.5374	0.5549	0.5768
1.8000	0.5308	0.5354	0.5537	0.5767
1.9000	0.5292	0.5339	0.5529	0.5768
2.0000	0.5279	0.5328	0.5525	0.5771

Appendix D

Appendix D shows the commented MATLAB code used to create the length, width, and area vectors of the nozzle being analyized.

```
function [AreaVector,LengthVector,Width,AStar,dx] = NozzleGeometry(Index)
%This function produces the useful information for a 15 degree conical
%   nozzle of length 3um. The Throat is currently 100nm with a constant
%   depth of 125nm.
%
%--------------------------------Inputs------------------------------------
%Index is the number of cells that the nozzle will be broken into. The
%larger this number, the finer the divisions in the nozzle will be.
%
%-------------------------------Outputs------------------------------------
%
%AreaVector is a vector of the areas as you progress down the nozzle. The
%first index in this vector should be the throat area, while the last index
%is the area at the exit of the nozzle. All entries are in m^2
%
%LengthVector is the vector that contains the position from the throat.
%This matches up to the AreaVector. The first entry is zero as it is the
%throat. Units are in m.
%
%Width is the width of the nozzle at any given distance fromt the throat of
%the nozzle. Units are in m.
%
%Astar is the area of the throat of the nozzle in units of m^2.
```

```
%
%dx is the distnace in meters between each index of the LengthVector.

%% ----------------------Constants and Parameters--------------------------

%key properties for the nozzzle's geometry
Angle = 15; %degrees
Length = 3000*10^-9;%m
depth = 125*10^-9; %m
WidthStar = 100*10^-9; %m^2

%% ------------------------Devrives Parameters-----------------------------

% This section defines the output properties given the constants listed
% before.
AStar = WidthStar*depth;
LengthVector = linspace(0,Length,Index);
Width = sind(Angle)*LengthVector+WidthStar;
AreaVector = Width.*depth;
dx = Length/length(LengthVector);
```

Appendix E

Appendix E conatins the code and data used in solving both solution methods for the shock-plug nozzle problem as well as an implimentation using the shock wave relations. This function, FixedThicknessShockPlugSolver.m , calculates the inlet and exit flow parameters, back pressure, sidewall pressure, area ratio, and entropy generated across a shock-plug with a given location, thickness, and phi parameter.

```
function [Mx,Px,Tx,My,Py,Ty,Pb,SidePressure,r,Sgen] =
ShockPlugSolver(To,Po,Gamma,R,AStar,AreaVector,LengthVector,ShockBackLocation,
Phi,ShockLength)
%[Mx,Px,Tx,My,Py,Ty,Pb,SidePressure,r,Sgen] =
ShockPlugSolver(To,Po,Gamma,R,Cp,AStar,AreaVector,LengthVector,Muo,Tref,S,Shoc
kLocation,Phi)
%This function solves for the flow properties before and after a shockplug
%as well as the Phi, Side pressure and area rati, r.
%
%----------------------------Inputs----------------------------------------
%
%To is the total temperature of the flow when on the leading edge of the
%shockwave in Kelvin
%
%Po is the total pressure of the flow on the leading edge of the shockave
%in Pa.
%
%Gamma is the raio of specific heats. This is normally 1.4, but is left as
%an input for the sake of robustness.
%
```

```
%R is the specific gas constant of the working fluid in the flow. This is
%in units of J/kg-K
%
%Astar is the area of the throat of the nozzle in units of m^2.
%
%AreaVector is a vector of the areas as you progress down the nozzle. The
%first index in this vector should be the throat area, while the last index
%is the area at the exit of the nozzle. All entries are in m^2
%
%LengthVector is the vector that contains the position from the throat.
%This matches up to the AreaVector. The first entry is zero as it is the
%throat.
%
%ShockBackLocation is an interger number that indexes where the shock's
% trailing edge is located in the LengthVector.
%
%Phi is a unitless parameter from zero to one that solves for the side
%pressure of the shockplug.
%
%-------------------------------Outputs-------------------------------------
%
%Mx is the Mach number just before the shockwave
%
%Px is the pressure just before the shockwave in Pa.
%
%Tx is the temperture just before the shockwave in K.
%
%My is the Mach number just after the shockwave
%
```

```
%Py is the pressure just after the shockwave in Pa.
%
%Ty is the temperture just after the shockwave in K.
%
%Pb is the pressure at the exit of the nozzle in Pa.
%
%SidePressure is the pressure that is present that is normal to the
%nozzle's boundry inside of the shockplug.
%
%r is the area ratio of the nozzle section that is being analyized. defined
%as the area of the leading edge of the shockplug over the trailing edge of
%the shock plug.
%
%Sgen is the entropy generated by the shock-plug in J/kg-K

%% -----------------------Constants and Variables--------------------------

%This is a set of constants that are used to shorten the following
%equations. These come from reacuring parts of the shock relations.
A = sqrt(Gamma/(To*R))*Po;
B = 0.5*(Gamma-1);
C = -(Gamma+1)/(2*Gamma-2);

%This calculation determines the mass flow rate of the system. This is done
%at the throat of the nozzle where M=1
MassFlow = A*AStar*(1+B)^C;

%% -------------------------Inlet Conditions-------------------------------
```

```
%Finds the index closest to the leading edge of the shock plug
[FILLER,ShockLocation] = min(abs(LengthVector-
(LengthVector(ShockBackLocation)-ShockLength)));
%The resulting Area Ratio
r = AreaVector(ShockLocation)/AreaVector(ShockBackLocation);

%This section solves for the inlet Mach number of the shock based on
%isentropic area ratio relations derived by Anderson. It then goes on to
%find the flow properties at this location.
    syms M real
        AreaRatio = (1/M^2*((2/(Gamma+1)*(1+B*M^2))^((Gamma+1)/(Gamma-
1))))^0.5*AStar == AreaVector(ShockLocation);
    SOLUTIONS = (solve(AreaRatio,M));

%Machx is the speed of the air at the leading edge of the shockplug.
Mx = max(double(SOLUTIONS));

%Static properties at the leading edge of the shockplug.
Tx = To/(1+((Gamma-1)/2)*Mx^2); %K
Px = Po/((1+(0.5*Gamma-0.5)*Mx^2)^(Gamma/(Gamma-1))); %Pa

%% ----------------------Shock Plug Solver---------------------------------

%AS A REMINDER
%A = sqrt(To*Gamma/R)*Po;
```

```
%B = 0.5*(Gamma-1);

%C = (3*Gamma-1)/(2*Gamma-2);


%This is system of shock-plug relations solve for the Mach number, pressure

%,and temperature at the trailing edge of the shock.

    syms MExit PExit TExit real


        CoMEqn = MExit/Mx == r*(Px/PExit)*sqrt(TExit/Tx);

        CoLMEqn = PExit/Px == (r*(1+Gamma*Mx^2)+(1-Phi)*(1-
r))/((1+Gamma*MExit^2)-Phi*(1-r));

        CoEEqn = TExit/Tx == (1+Mx^2*((Gamma-1)/2))/(1+MExit^2*((Gamma-1)/2));


%This section solves the system of equations and coverts the answer into a

%double for use later.

SOLUTIONS = (solve(CoLMEqn,CoMEqn,CoEEqn,MExit,PExit,TExit));


[My,Index] = min(double(SOLUTIONS.MExit));

PExitInter = double(SOLUTIONS.PExit);

Py = PExitInter(Index); %Pa

TyInter = double(SOLUTIONS.TExit);

Ty = TyInter(Index); %K


%The total properties after the shock plug are now found.

Tyo = Ty*(1+B*My^2); %K

Pyo = Py*(1+B*My^2)^(Gamma/(Gamma-1)); %Pa


%This is the explicit solution for the side pressure.

SidePressure = Px+Phi*(Py-Px); %Pa
```

```
%The entropy generated from the shock plug solution.
Sgen = -R*log(Pyo/Po); %J/kg-K

%% --------------------------Exit Conditions-------------------------------

%Once the properties at the trailing edge of the shockplug are known, the
%back pressure can be found. This is done in the same way as the Shockwave
%Solver function.

    syms M
        CoMEqn = (sqrt(Gamma/(R*Tyo))*Pyo*max(AreaVector)*M)*(1+B*M^2)^C ==
MassFlow;
    MExit = min((double(vpasolve(CoMEqn,M,[0 3]))));

Pb = Pyo/((1+B*MExit^2)^(Gamma/(Gamma-1))); %Pa
```

The FixedThicknessNoPhiShockPlugSolver.m function operates similar to the previous function, but it does not use the shock-plug reations. Instead it uses a convervation equations system with no side pressure term in the conservation of linear momentum equation. With the acception of phi and side pressure, this function has the same inputs and output of FixedThicknessShockPlugSolver.m.

```
function [Mx,Px,Tx,My,Py,Ty,Pb,r,Sgen] =
NoPhiShockPlugSolver(To,Po,Gamma,R,Cp,AStar,AreaVector,LengthVector,ShockBackL
ocation,ShockLength)
%[Mx,Px,Tx,My,Py,Ty,Pb,SidePressure,r,Sgen] =
ShockPlugSolver(To,Po,Gamma,R,Cp,AStar,AreaVector,LengthVector,Muo,Tref,S,Shoc
kLocation,Phi)
%This function solves for the flow properties before and after a shockplug
%as well as the Phi, Side pressure and area rati, r.
%
%----------------------------Inputs---------------------------------------
%
%To is the total temperature of the flow when on the leading edge of the
%shockwave in Kelvin
%
%Po is the total pressure of the flow on the leading edge of the shockave
%in Pa.
%
%Gamma is the raio of specific heats. This is normally 1.4, but is left as
%an input for the sake of robustness.
%
%R is the specific gas constant of the working fluid in the flow. This is
%in units of J/kg-K
```

```
%
%Cp is the specific heat with constant pressure of the working fluid. This
%is inJ/kg-k
%
%Astar is the area of the throat of the nozzle in units of m^2.
%
%AreaVector is a vector of the areas as you progress down the nozzle. The
%first index in this vector should be the throat area, while the last index
%is the area at the exit of the nozzle. All entries are in m^2
%
%LengthVector is the vector that contains the position from the throat.
%This matches up to the AreaVector. The first entry is zero as it is the
%throat.
%
%ShockBackLocation is an interger number that indexes where the shock's
%trailing edge is located in the LengthVector.
%
%------------------------------Outputs-------------------------------------
%
%Mx is the Mach number just before the shockwave
%
%Px is the pressure just before the shockwave in Pa.
%
%Tx is the temperture just before the shockwave in K.
%
%My is the Mach number just after the shockwave
%
%Py is the pressure just after the shockwave in Pa.
%
```

```
%Ty is the temperture just after the shockwave in K.
%
%Pb is the pressure at the exit of the nozzle in Pa.
%
%r is the area ratio of the nozzle section that is being analyized. defined
%as the area of the leading edge of the shockplug over the trailing edge of
%the shock plug.
%
%Sgen is the entropy generated by the shock-plug in J/kg-K
%% -----------------------Constants and Variables--------------------------

%This is a set of constants that are used to shorten the following
%equations. These come from reacuring parts of the shock relations.
A = sqrt(Gamma/(To*R))*Po;
B = 0.5*(Gamma-1);
C = -(Gamma+1)/(2*Gamma-2);

%This calculation determines the mass flow rate of the system. This is done
%at the throat of the nozzle where M=1
MassFlow = A*AStar*(1+B)^C;

%% -------------------------Inlet Conditions-------------------------------

%Finds the index closest to the leading edge of the shock plug
[FILLER,ShockLocation] = min(abs(LengthVector-
(LengthVector(ShockBackLocation)-ShockLength)));
%Area Ratio
r = AreaVector(ShockLocation)/AreaVector(ShockBackLocation);
```

```
%This section solves for the inlet Mach number of the shock based on
%isentropic area ratio relations derived by Anderson. It then goes on to
%find the flow properties at this location.
    syms M real
        AreaRatio = (1/M^2*((2/(Gamma+1)*(1+B*M^2))^((Gamma+1)/(Gamma-
1))))^0.5*AStar == AreaVector(ShockLocation);
    SOLUTIONS = (solve(AreaRatio,M));

%Machx is the speed of the air at the leading edge of the shockplug.
Mx = max(double(SOLUTIONS));

%Static properties at the leading edge of the shockplug.
Tx = To/(1+((Gamma-1)/2)*Mx^2); %K
Px = Po/((1+(0.5*Gamma-0.5)*Mx^2)^(Gamma/(Gamma-1))); %Pa

%% ----------------------Shock Plug Solver---------------------------------

%AS A REMINDER
%A = sqrt(To*Gamma/R)*Po;
%B = 0.5*(Gamma-1);
%C = (3*Gamma-1)/(2*Gamma-2);

%Since the system is SS and Adiabatic with no work, the total enthalpy is
%the only source of energy. Similarly the momentum into the system is also
%equal to the momentum out.
Ho = Cp*To;
MomentumIn = Px*AreaVector(ShockLocation)+ MassFlow*Mx*sqrt(Gamma*R*Tx);
```

```
%The system of Mass, Momentum,and Energy solves for the Mach number,
%pressure, and temperature at the trailing edge of the shock, as well as,
%the phi parameter.
    syms Me PExit TExit real
        CoMEqn = PExit*AreaVector(ShockBackLocation)*Me*sqrt(Gamma/(R*TExit))
== MassFlow;

        PA2 = PExit*AreaVector(ShockBackLocation);
        SidePressure = 0; %Px + Phi*(PExit-Px);
        MdotV = MassFlow*Me*sqrt((Gamma*R*TExit));
        CoLMEqn = PA2+MdotV-SidePressure*(AreaVector(ShockBackLocation)-
AreaVector(ShockLocation))== MomentumIn;

        CoEEqn = (Me^2*(Gamma*R*TExit))/2+Cp*TExit == Ho;

%This section solves the system of equations and coverts the answer into a
%double for use later.
SOLUTIONS = (solve(CoLMEqn,CoMEqn,CoEEqn,Me,PExit,TExit));

[My,Index] = min(double(SOLUTIONS.Me));
PExitInter = double(SOLUTIONS.PExit);
Py = PExitInter(Index); %Pa
TyInter = double(SOLUTIONS.TExit);
Ty = TyInter(Index); %K

%The total properties after the shock plug are now found.
Tyo = Ty*(1+B*My^2); %K
```

```
Pyo = Py*(1+B*My^2)^(Gamma/(Gamma-1)); %Pa

%The entropy generated from the shock plug solution.
Sgen = -R*log(Pyo/Po); %J/kg-K

%% --------------------------Exit Conditions-------------------------------

%Once the properties at the trailing edge of the shockplug are known, the
%back pressure can be found. This is done in the same way as the Shockwave
%Solver function.

    syms M
        CoMEqn = (sqrt(Gamma/(R*Tyo))*Pyo*max(AreaVector)*M)*(1+B*M^2)^C ==
MassFlow;
    MExit = min((double(vpasolve(CoMEqn,M,[0 3]))));

Pb = Pyo/((1+B*MExit^2)^(Gamma/(Gamma-1))); %Pa
```

The ShockwaveSolver.m function used the standard shock wave relations to find the inlet and exit conditions, as well as, back pressure and entropy generation for a shock wave at a given location.

```
function
[Mx,Px,Tx,My,Py,Ty,Pb,Sgen]=ShockwaveSolver(To,Po,Gamma,R,AStar,AreaVector,Sho
ckLocation)
%[Mx,Px,Tx,My,Py,Ty,Pb]=ShockwaveSolver(To,Po,Gamma,R,Cp,AStar,AreaVector,Leng
thVector,Muo,Tref,S,ShockLocation)
%This function solves the normal shok reations from Anderson given the
%properties of the flow and nozzle geometry.
%
%----------------------------Inputs----------------------------------------
%
%To is the total temperature of the flow when on the leading edge of the
%shockwave in Kelvin
%
%Po is the total pressure of the flow on the leading edge of the shockave
%in Pa.
%
%Gamma is the raio of specific heats. This is normally 1.4, but is left as
%an input for the sake of robustness.
%
%R is the specific gas constant of the working fluid in the flow. This is
%in units of J/kg-K
%
%Cp is the specific heat with constant pressure of the working fluid. This
%is inJ/kg-k
%
```

```
%Astar is the area of the throat of the nozzle in units of m^2.
%
%AreaVector is a vector of the areas as you progress down the nozzle. The
%first index in this vector should be the throat area, while the last index
%is the area at the exit of the nozzle. All entries are in m^2
%
%LengthVector is the vector that contains the position from the throat.
%This matches up to the AreaVector. The first entry is zero as it is the
%throat.
%
%Muo is the reference mu value for the Sutherland Equation to calculate the
%viscosity of the flow at any point.
%
%Tref is the reference temperature value for the Sutherland Equation to
%calculate the viscosity of the flow at any point.
%
%S is another reference value for the Sutherland Equation to calculate the
%viscosity of the flow at any point.
%
%ShockLocation is an interger number that indexes where the shock is
%located in the LengthVector.
%
%------------------------------Outputs------------------------------------
%
%Mx is the Mach number just before the shockwave
%
%Px is the pressure just before the shockwave in Pa.
%
%Tx is the temperture just before the shockwave in K.
```

```
%
%My is the Mach number just after the shockwave
%
%Py is the pressure just after the shockwave in Pa.
%
%Ty is the temperture just after the shockwave in K.
%
%Pb is the pressure at the exit of the nozzle in Pa.

%% -----------------------Constants and Variables--------------------------

%This is a set of constants that are used to shorten the following
%equations. These come from reacuring parts of the shock relations.
A = sqrt(Gamma/(To*R))*Po;
B = 0.5*(Gamma-1);
C = -(Gamma+1)/(2*Gamma-2);

%This calculation determines the mass flow rate of the system. This is done
%at the throat of the nozzle where M=1. This is held constant throughout
%the nozzle due to there being one inlet and exit as well as the steady
%state assumption.
MassFlow = A*AStar*(1+B)^C;

%% ------------------------Symbolic Equation-------------------------------

%This symbolic equation used the isentropic relations derived by Anderson
%to determine the Mach number just before te shockwave.
    syms M real
```

```
        AreaRatio = (1/M^2*((2/(Gamma+1))*((1+B*M^2)))^(-2*C))^0.5*AStar ==
AreaVector(ShockLocation);
    Mx = max(double(solve(AreaRatio,M)));

%% -------------------------Shock Relations--------------------------------



%This section uses the normal shock relations derived by Anderson to
%determine the static and stagnation properties of the flow before and
%after the shockwave.
Px = Po./((1+(B.*(Mx.^2))).^(Gamma/(Gamma-1))); %Pa
Tx = To./(1+(B.*(Mx.^2))); %K
My = sqrt((1+B.*Mx.^2)./(Gamma.*Mx.^2-B));
Py = Px.*(1+((2*Gamma)/(Gamma+1)).*(Mx.^2-1)); %Pa
Ty = Tx.*(1+((2*Gamma)/(Gamma+1)).*(Mx.^2-1)).*((2+(Gamma-
1)*Mx.^2)./((Gamma+1)*Mx.^2)); %K
Tyo = Ty*(1+B*My^2); %K
Pyo = Py*(1+B*My^2)^(Gamma/(Gamma-1)); %Pa

%This symbolic equation solves the CoM to determine the Mach number at the
%exit of the nozzle.
    syms Me real
        CoMEqn = (sqrt(Gamma/(R*Tyo))*Pyo*max(AreaVector)*Me)*((1+B*Me^2)^C)==
MassFlow;
    MExit = min((double(vpasolve(CoMEqn,Me,[0 3]))));

%Since the Mach number of the exit of the nozzle is known, the back
%pressure is calculated by Anderson's relations.
```

```
Pb = Pyo/((1+B*MExit^2)^(Gamma/(Gamma-1)));


Sgen = -R*log(Pyo/Po);
```

Appendix F

Appendix F conatins the code and data used to create the surface plots in Figures 4.5 through 4.10. This Appendix also inludes the contour plots of each surface plot. The code titled ShockComparison.m is the main program for creating the data in this appendix, although it also uses the functions ShockwaveSolver.m and FixedThicknessLeadingEdgeShockPlugSolver.m as shown in Appendix E.

```
%ShockComparision.m
%
%This program finds the location and flow propteries of a shockwave and
%shock and shockplug. It uses the information from the NozzleGeometry
%function, was well as several flow properties. This program finds the
%location of both shocks by solving for a fixed back pressure.
%
% -----------------------Function Passthrough Inputs-----------------------
%
%To is the total temperature of the flow when on the leading edge of the
%shockwave in Kelvin
%
%Po is the total pressure of the flow on the leading edge of the shockave
%in Pa.
%
%Gamma is the raio of specific heats. This is normally 1.4, but is left as
%an input for the sake of robustness.
%
%R is the specific gas constant of the working fluid in the flow. This is
%in units of J/kg-K
%
```

```
%Cp is the specific heat with constant pressure of the working fluid. This
%is inJ/kg-k
%
%Index is the number of cells in the nozzle geometry outputs. A higher
%number leads to a higher resolution of the nozzle.
%
%Muo is the reference mu value for the Sutherland Equation to calculate the
%viscosity of the flow at any point.
%
%Tref is the reference temperature value for the Sutherland Equation to
%calculate the viscosity of the flow at any point.
%
%S is another reference value for the Sutherland Equation to calculate the
%viscosity of the flow at any point.
%
%ShockLocWave/Plug is an interger number that indexes where the shock is
%located in the LengthVector. The -Wave and -Plug suffixes differentiate
%between the type of shock.
%
%Phi is a unitless parameter from zero to one that solves for the side
%pressure of the shockplug. In practice, these bounds do not hold.
%
% ----------------------Function Passthrough Outputs-----------------------
%
%Astar is the area of the throat of the nozzle in units of m^2.
%
%AreaVector is a vector of the areas as you progress down the nozzle. The
%first index in this vector should be the throat area, while the last index
%is the area at the exit of the nozzle. All entries are in m^2
```

```
%
%LengthVector is the vector that contains the position from the throat.
%This matches up to the AreaVector. The first entry is zero as it is the
%throat.
%
%MxWave/Plug is the Mach number just before the shockwave. The -Wave and
%-Plug suffixes differentiate between the type of shock.
%
%PxWave/Plug is the pressure just before the shockwave in Pa.The -Wave and
%-Plug suffixes differentiate between the type of shock.
%
%TxWave/Plug is the temperture just before the shockwave in K.The -Wave and
%-Plug suffixes differentiate between the type of shock.
%
%MyWave/Plug is the Mach number just after the shockwaveThe -Wave and
%-Plug suffixes differentiate between the type of shock.
%
%PyWave/Plug is the pressure just after the shockwave in Pa.The -Wave and
%-Plug suffixes differentiate between the type of shock.
%
%TyWave/Plug is the temperture just after the shockwave in K.The -Wave and
%-Plug suffixes differentiate between the type of shock.
%
%PbWave/Plug is the pressure at the exit of the nozzle in Pa at the guessed
%shock location.The -Wave and -Plug suffixes differentiate between the type
%of shock.
%
%SidePressure is the pressure that is present that is normal to the
%nozzle's boundry inside of the shockplug.
```

```
%
%r is the area ratio of the nozzle section that is being analyized. defined
%as the area of the leading edge of the shockplug over the trailing edge of
%the shock plug.

%% Initialization Code
%clears prior data
clc
clear variables
close all

%% -----------------------Constants and Variables--------------------------

%These are gas and flow properties used throughout the code.
Po = 101.3*10^3; %Pa
To = 298; %K
Gamma = 1.4;
R = 286.9; %J/kg K Gas Constant of Air
Cp = 1005.; %J/kg K Specific Heat of Air at 20C
d = 3.711*10^-10; %m diameter of air molecules
kB = 1.3806488*10^-23; %JK Boltzmann constant
A = (Gamma-1)/2;
Pb = 75*10^3; %Set back pressure in Pa

%% -------------------------Nozzle Geometry-------------------------------

%This section calls the NozzleGeometry function and produces length, area
%and width vectors of the nozzle.
```

```
Index = 100000;
[AreaVector,LengthVector,Width,AStar,dx] = NozzleGeometry(Index);

%% ------------------------ShockWave Solution------------------------------

%This section uses an inital guess of the location of the shockwave to
%produce a back prressure. This back pressure is compared to the desired
%back pressure, set in the Constants and Variables section. A new location
%is calcualted then fed back into the function to get a new back pressure.
%This contuinues until the difference between the calculated and guessed
%back pressure is within .1%.

%This section creates the initial parameters used to find the correct
shockwave location.
Error = Pb;
i=1;
ShockLocWave = 8000;
DShockLocWave = 1;

%This loop solves for the location of the shockwave that matches the back
%pressure to within 10 Pa.
while abs(Error) > 10
    i


[MxWave,PxWave,TxWave,MyWave,PyWave,TyWave,PbWave,SgenWave]=ShockwaveSolver(To
,Po,Gamma,R,AStar,AreaVector,ShockLocWave);
```

```
%Calling ShockwaveSolver at a small step to create and estimate of the first
derivative of the back pressure with respect to the shock wave location.
[DMxWave,DPxWave,DTxWave,DMyWave,DPyWave,DTyWave,DPbWave,DSgenWave]=ShockwaveS
olver(To,Po,Gamma,R,AStar,AreaVector,ShockLocWave+DShockLocWave);

%This is a simple root solving method to solve for the correct location.
    dPdShockLoc = (DPbWave-PbWave)/DShockLocWave;
    Error = (Pb-PbWave)
    ShockLocWave = ShockLocWave + int16(Error/dPdShockLoc);
    i = i+1;
end

%The minimum value for the Phi value at the shock wave conditions.
PhiminWave = -PxWave/(PyWave-PxWave);

%% ------------------------Shock-plug Solution-----------------------------

%This section creates two arrays, one for the Phi and Thickness variables.
%These arrays determine the range and values that the code will run over to
%create the surface plots.

%Creation of the vectors that represent the values of Phi and thickness that
is going to be looped over.
PhiLength = 23;
MaxPhi = 3.4;
MinPhi = -1.0;
Phi = linspace(MinPhi,MaxPhi,PhiLength);
ChangeInPhi = (max(Phi)-min(Phi))/PhiLength;
```

```
ThicknessLength = 21;
MaxThickness = 1*10^-7;
MinThickness = 0;
Thickness = linspace(MinThickness,MaxThickness,ThicknessLength);
ChangeInThickness = (MaxThickness-MinThickness)/ThicknessLength;

%Initialization code for the data created in the following loop.
MxPlug = zeros(length(Phi),length(Thickness));
PxPlug = zeros(length(Phi),length(Thickness));
TxPlug = zeros(length(Phi),length(Thickness));
MyPlug = zeros(length(Phi),length(Thickness));
PyPlug = zeros(length(Phi),length(Thickness));
TyPlug = zeros(length(Phi),length(Thickness));
PbPlug = zeros(length(Phi),length(Thickness));
SidePressure = zeros(length(Phi),length(Thickness));
r = zeros(length(Phi),length(Thickness));
Sgen = zeros(length(Phi),length(Thickness));

%This loop goes over the values set up in the Phi and Thickness variables
%and places them in matrices.
for i = 1:length(Phi)
    for j = 1:length(Thickness)
        i
        j

[MxPlug(i,j),PxPlug(i,j),TxPlug(i,j),MyPlug(i,j),PyPlug(i,j),TyPlug(i,j),PbPlu
g(i,j),SidePressure(i,j),r(i,j),Sgen(i,j)] =
```

```
FixedThicknessLeadingEdgeShockPlugSolver(To,Po,Gamma,R,AStar,AreaVector,Length
Vector,ShockLocWave,Phi(i),Thickness(j));
    end
end

%Saving all of the results from the shockplug solver so that this code does
%not need to be run everytime we need a new graph.
save ComparisonThickness.mat Thickness
save ComparisonPhi.mat Phi
save ComparisonMy.mat MyPlug
save ComparisonPy.mat PyPlug
save ComparisonTy.mat TyPlug
save ComparisonBackPressure.mat PbPlug
save ComparisonSidePressure.mat SidePressure
save Comparisonr.mat r
save ComparisonSGen.mat Sgen

%This matrices are used to compare the entropy generation to the entropy
%generated by the shock plug analysis.
Isentropic = zeros(size(Sgen));
ShockwaveEntropy = SgenWave*ones(size(Sgen));

%% ---------------------------Surface Plots--------------------------------

%Various plots of the data.
surf(r(1,:),Phi,Sgen);
% hold on
% surf(r(1,:),Phi,Isentropic,gradient());Isentropic
```

```
ylabel('Phi');
xlabel('Area Ratio')
zlabel('Entropy Generation (kJ/kg-K)')
axis tight

figure
contour(r(1,:),Phi,Sgen,20,'ShowText','on');
ylabel('Phi')
xlabel('Area Ratio')

figure
surf(r(1,:),Phi,MyPlug);
zlabel('Exit Mach Number')
ylabel('Phi')
xlabel('Area Ratio')
% hold on
% surf(r(1,:),Phi,MyWave*ones(size(MyPlug)));

figure
contour(r(1,:),Phi,MyPlug,20,'ShowText','on');
ylabel('Phi')
xlabel('Area Ratio')

figure
surf(r(1,:),Phi,PyPlug);
zlabel('Exit Pressure')
ylabel('Phi')
```

```
xlabel('Area Ratio')
% hold on
% surf(r(1,:),Phi,PyWave*ones(size(PyPlug)));

figure
contour(r(1,:),Phi,PyPlug,20,'ShowText','on');
ylabel('Phi')
xlabel('Area Ratio')

figure
surf(r(1,:),Phi,TyPlug);
zlabel('Exit Temperature')
ylabel('Phi')
xlabel('Area Ratio')
% hold on
% surf(r(1,:),Phi,TyWave*ones(size(TyPlug)));

figure
contour(r(1,:),Phi,TyPlug,20,'ShowText','on');
ylabel('Phi')
xlabel('Area Ratio')

figure
surf(r(1,:),Phi,PbPlug);
zlabel('Back Pressure')
ylabel('Phi')
xlabel('Area Ratio')
% hold on
```

```
% surf(r(1,:),Phi,Pb*ones(size(PbPlug)));


figure
contour(r(1,:),Phi,PbPlug,20,'ShowText','on');
ylabel('Phi')
xlabel('Area Ratio')


figure
surf(r(1,:),Phi,SidePressure);
zlabel('Side Pressure')
ylabel('Phi')
xlabel('Area Ratio')


figure
contour(r(1,:),Phi,SidePressure,20,'ShowText','on');
ylabel('Phi')
xlabel('Area Ratio')
```

The following tables are the numerical values used to create the figures in Section 4.5. The contour plot of the data is shown just below its corrisponding table.

Table F.1.a: Exit Mach Number a Shock-plug with for values of Phi from -1.00 to 3.40, and Area Ratio from 1.000 to 0.9259

Area Ratio/Phi	1.0000	0.9921	0.9843	0.9766	0.9690	0.9616	0.9542	0.9470	0.9399	0.9329	0.9259
-1.00	0.5863	0.5903	0.5941	0.5978	0.6014	0.6048	0.6081	0.6113	0.6144	0.6173	0.6201
-0.80	0.5863	0.5891	0.5918	0.5945	0.5970	0.5993	0.6016	0.6038	0.6059	0.6079	0.6098
-0.60	0.5863	0.5880	0.5896	0.5911	0.5925	0.5939	0.5951	0.5964	0.5975	0.5986	0.5996
-0.40	0.5863	0.5868	0.5873	0.5877	0.5881	0.5884	0.5887	0.5889	0.5891	0.5893	0.5894
-0.20	0.5863	0.5857	0.5850	0.5844	0.5837	0.5830	0.5823	0.5815	0.5808	0.5801	0.5793
0.00	0.5863	0.5845	0.5828	0.5810	0.5793	0.5776	0.5759	0.5742	0.5725	0.5709	0.5692
0.20	0.5863	0.5834	0.5805	0.5777	0.5749	0.5722	0.5695	0.5669	0.5643	0.5617	0.5592
0.40	0.5863	0.5822	0.5782	0.5743	0.5705	0.5668	0.5632	0.5596	0.5561	0.5527	0.5493
0.60	0.5863	0.5811	0.5760	0.5710	0.5662	0.5615	0.5568	0.5523	0.5479	0.5436	0.5394
0.80	0.5863	0.5799	0.5737	0.5677	0.5618	0.5561	0.5505	0.5451	0.5398	0.5347	0.5296
1.00	0.5863	0.5788	0.5715	0.5644	0.5575	0.5508	0.5443	0.5379	0.5318	0.5257	0.5198
1.20	0.5863	0.5776	0.5693	0.5611	0.5532	0.5455	0.5380	0.5308	0.5237	0.5168	0.5101
1.40	0.5863	0.5765	0.5670	0.5578	0.5489	0.5402	0.5318	0.5236	0.5157	0.5080	0.5005
1.60	0.5863	0.5753	0.5648	0.5545	0.5446	0.5350	0.5256	0.5166	0.5078	0.4992	0.4909
1.80	0.5863	0.5742	0.5626	0.5512	0.5403	0.5298	0.5195	0.5095	0.4999	0.4905	0.4813
2.00	0.5863	0.5730	0.5603	0.5480	0.5360	0.5245	0.5133	0.5025	0.4920	0.4818	0.4718
2.20	0.5863	0.5719	0.5581	0.5447	0.5318	0.5193	0.5072	0.4955	0.4842	0.4731	0.4624
2.40	0.5863	0.5708	0.5559	0.5415	0.5275	0.5142	0.5011	0.4885	0.4764	0.4645	0.4530
2.60	0.5863	0.5696	0.5537	0.5382	0.5233	0.5090	0.4951	0.4816	0.4686	0.4560	0.4437
2.80	0.5863	0.5685	0.5515	0.5350	0.5191	0.5038	0.4890	0.4747	0.4609	0.4475	0.4344
3.00	0.5863	0.5673	0.5492	0.5317	0.5149	0.4987	0.4830	0.4678	0.4532	0.4390	0.4252
3.20	0.5863	0.5662	0.5470	0.5285	0.5107	0.4936	0.4770	0.4610	0.4456	0.4306	0.4160
3.40	0.5863	0.5651	0.5448	0.5253	0.5065	0.4885	0.4711	0.4542	0.4380	0.4222	0.4069

Table F.1.b: Exit Mach Number for a Shock-plug with values of Phi from -1.00 to 3.40, and Area Ratio from 0.9192 to 0.8621

Area Ratio/Phi	0.9192	0.9124	0.9058	0.8993	0.8929	0.8866	0.8803	0.8742	0.8681	0.8621
-1.00	0.6228	0.6253	0.6278	0.6301	0.6324	0.6345	0.6365	0.6384	0.6403	0.6420
-0.80	0.6116	0.6133	0.6149	0.6165	0.6179	0.6193	0.6206	0.6219	0.6230	0.6241
-0.60	0.6005	0.6014	0.6022	0.6029	0.6036	0.6043	0.6049	0.6054	0.6059	0.6063
-0.40	0.5895	0.5895	0.5895	0.5895	0.5894	0.5893	0.5892	0.5890	0.5889	0.5886
-0.20	0.5785	0.5777	0.5769	0.5761	0.5753	0.5745	0.5736	0.5728	0.5719	0.5711
0.00	0.5676	0.5660	0.5644	0.5628	0.5613	0.5597	0.5582	0.5567	0.5551	0.5536
0.20	0.5568	0.5544	0.5520	0.5497	0.5473	0.5451	0.5428	0.5406	0.5385	0.5363
0.40	0.5460	0.5428	0.5396	0.5365	0.5335	0.5305	0.5276	0.5247	0.5219	0.5191
0.60	0.5353	0.5313	0.5274	0.5235	0.5198	0.5161	0.5125	0.5089	0.5054	0.5020
0.80	0.5247	0.5199	0.5152	0.5106	0.5061	0.5017	0.4974	0.4932	0.4891	0.4851
1.00	0.5141	0.5085	0.5031	0.4978	0.4925	0.4874	0.4825	0.4776	0.4729	0.4682
1.20	0.5036	0.4973	0.4910	0.4850	0.4791	0.4733	0.4677	0.4622	0.4568	0.4515
1.40	0.4932	0.4860	0.4791	0.4723	0.4657	0.4592	0.4530	0.4468	0.4408	0.4349
1.60	0.4828	0.4749	0.4672	0.4598	0.4524	0.4453	0.4384	0.4316	0.4249	0.4184
1.80	0.4725	0.4638	0.4554	0.4473	0.4392	0.4314	0.4239	0.4164	0.4092	0.4021
2.00	0.4622	0.4528	0.4437	0.4348	0.4261	0.4177	0.4095	0.4014	0.3935	0.3859
2.20	0.4521	0.4419	0.4321	0.4225	0.4131	0.4040	0.3952	0.3865	0.3780	0.3698
2.40	0.4419	0.4311	0.4205	0.4103	0.4002	0.3905	0.3810	0.3717	0.3626	0.3538
2.60	0.4318	0.4203	0.4090	0.3981	0.3874	0.3770	0.3669	0.3570	0.3473	0.3380
2.80	0.4218	0.4095	0.3976	0.3860	0.3747	0.3636	0.3529	0.3424	0.3322	0.3222
3.00	0.4119	0.3989	0.3862	0.3740	0.3620	0.3503	0.3390	0.3279	0.3171	0.3066
3.20	0.4020	0.3883	0.3749	0.3621	0.3494	0.3372	0.3253	0.3136	0.3022	0.2911
3.40	0.3921	0.3777	0.3637	0.3502	0.3370	0.3241	0.3116	0.2993	0.2873	0.2757

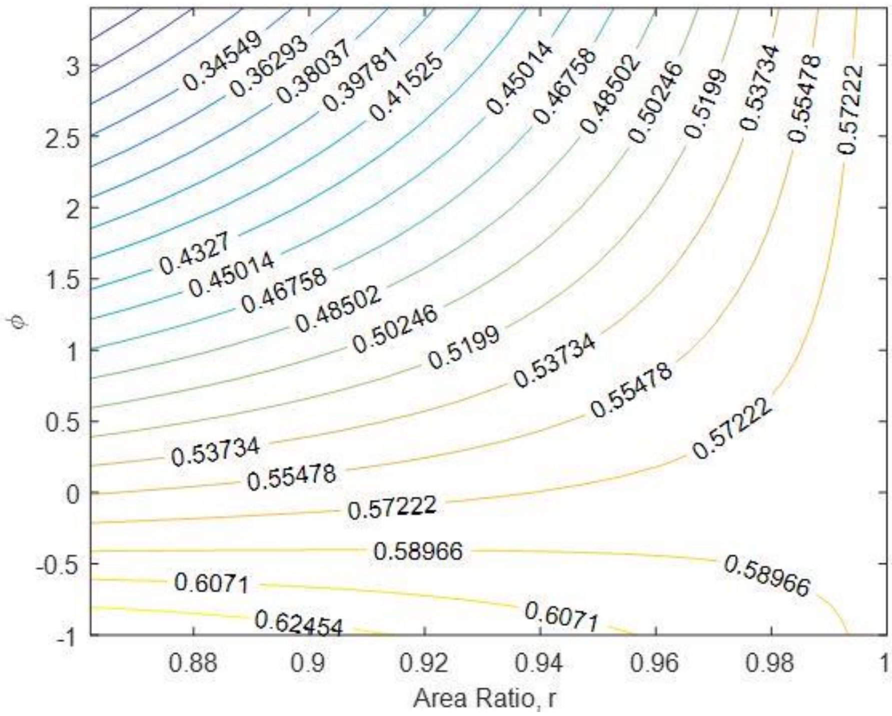

Figure F.1: Contour Plot of the Exit Mach Number with Respect to Phi and Area Ratio

Table F.2.a: Exit Pressure in Pa for values of Phi from -1.00 to 3.40, and Area Ratio from 1.000 to 0.9259.

Area Ratio/Phi	1.0000	0.9921	0.9843	0.9766	0.9690	0.9616	0.9542	0.9470	0.9399	0.9329	0.9259
-1.00	59759	58857	57995	57161	56357	55588	54842	54124	53435	52766	52121
-0.80	59759	58981	58233	57507	56806	56132	55476	54842	54232	53638	53063
-0.60	59759	59104	58473	57857	57260	56684	56121	55575	55047	54532	54030
-0.40	59759	59228	58714	58210	57719	57243	56777	56322	55881	55447	55025
-0.20	59759	59352	58956	58566	58183	57811	57444	57084	56733	56387	56047
0.00	59759	59477	59200	58925	58653	58387	58122	57861	57605	57350	57098
0.20	59759	59602	59445	59287	59129	58971	58813	58654	58497	58339	58180
0.40	59759	59727	59692	59652	59610	59564	59516	59464	59410	59353	59294
0.60	59759	59853	59940	60021	60097	60166	60231	60291	60345	60396	60441
0.80	59759	59979	60189	60393	60589	60777	60959	61135	61303	61466	61623
1.00	59759	60106	60440	60768	61088	61397	61701	61997	62284	62567	62842
1.20	59759	60232	60693	61147	61593	62027	62456	62878	63290	63698	64099
1.40	59759	60360	60947	61530	62104	62667	63226	63778	64321	64861	65396
1.60	59759	60487	61203	61915	62621	63316	64010	64699	65379	66059	66735
1.80	59759	60615	61460	62305	63145	63976	64809	65640	66464	67292	68119
2.00	59759	60744	61719	62698	63675	64646	65624	66602	67577	68561	69549
2.20	59759	60872	61979	63094	64212	65327	66454	67587	68721	69870	71028
2.40	59759	61002	62241	63495	64756	66019	67301	68595	69895	71219	72559
2.60	59759	61131	62505	63899	65307	66722	68164	69627	71102	72610	74144
2.80	59759	61261	62770	64307	65865	67437	69045	70683	72342	74046	75787
3.00	59759	61392	63037	64719	66430	68164	69945	71765	73618	75529	77491
3.20	59759	61522	63306	65134	67003	68903	70862	72874	74930	77061	79259
3.40	59759	61653	63576	65554	67583	69654	71799	74010	76281	78645	81095

Table F.2.b: Exit Pressure in Pa for values of Phi from -1.00 to 3.40, and Area Ratio from 0.9192 to 0.8621.

Area Ratio/Phi	0.9192	0.9124	0.9058	0.8993	0.8929	0.8866	0.8803	0.8742	0.8681	0.8621
-1.00	51502	50900	50318	49760	49216	48690	48184	47692	47214	46755
-0.80	52509	51969	51445	50940	50447	49969	49507	49056	48618	48195
-0.60	53546	53072	52610	52164	51727	51301	50889	50485	50091	49709
-0.40	54614	54211	53817	53435	53059	52691	52333	51981	51637	51302
-0.20	55716	55389	55068	54754	54445	54141	53844	53551	53262	52981
0.00	56852	56606	56364	56126	55889	55656	55426	55198	54972	54751
0.20	58023	57866	57708	57552	57396	57239	57084	56929	56774	56620
0.40	59233	59170	59104	59037	58968	58897	58824	58750	58675	58598
0.60	60483	60520	60554	60584	60610	60633	60653	60669	60683	60693
0.80	61774	61921	62062	62197	62328	62455	62576	62694	62807	62916
1.00	63110	63373	63630	63880	64126	64367	64602	64832	65059	65279
1.20	64491	64880	65263	65639	66011	66378	66738	67096	67449	67796
1.40	65922	66446	66965	67477	67988	68495	68995	69495	69992	70482
1.60	67404	68074	68741	69402	70066	70727	71383	72043	72701	73355
1.80	68940	69768	70596	71420	72251	73084	73914	74753	75595	76436
2.00	70534	71531	72534	73536	74552	75576	76602	77643	78693	79748
2.20	72189	73369	74562	75760	76980	78216	79460	80730	82018	83318
2.40	73909	75287	76686	78099	79545	81017	82508	84036	85595	87178
2.60	75697	77289	78914	80563	82260	83996	85763	87585	89456	91365
2.80	77558	79382	81253	83163	85137	87169	89249	91407	93635	95924
3.00	79496	81572	83713	85909	88193	90556	92991	95532	98173	100905
3.20	81517	83867	86302	88816	91444	94181	97019	100001	103121	106373
3.40	83625	86273	89033	91897	94911	98070	101367	104857	108537	112402

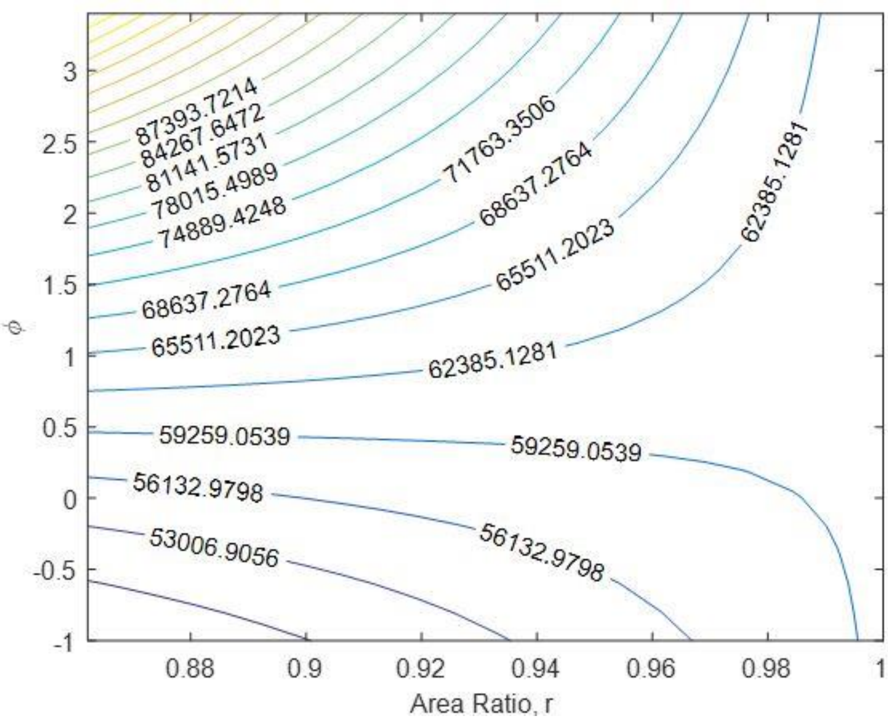

Figure F.2: Contour Plot of the Exit Pressure in Pa with Respect to Phi and Area Ratio

Table F.3.a: Exit Temperature for a Shock-plug in K for values of Phi from -1.00 to 3.40, and Area Ratio from 1.000 to 0.9259

Area Ratio/Phi	1.0000	0.9921	0.9843	0.9766	0.9690	0.9616	0.9542	0.9470	0.9399	0.9329	0.9259
-1.00	278.83	278.59	278.35	278.12	277.90	277.68	277.48	277.28	277.08	276.90	276.72
-0.80	278.83	278.66	278.49	278.33	278.17	278.03	277.88	277.75	277.62	277.49	277.37
-0.60	278.83	278.73	278.63	278.54	278.45	278.37	278.29	278.21	278.14	278.07	278.01
-0.40	278.83	278.80	278.77	278.74	278.72	278.70	278.68	278.67	278.66	278.65	278.64
-0.20	278.83	278.87	278.91	278.95	278.99	279.03	279.08	279.12	279.17	279.21	279.26
0.00	278.83	278.94	279.05	279.15	279.26	279.36	279.46	279.57	279.67	279.77	279.86
0.20	278.83	279.01	279.18	279.36	279.52	279.69	279.85	280.00	280.16	280.31	280.46
0.40	278.83	279.08	279.32	279.56	279.79	280.01	280.23	280.44	280.64	280.84	281.04
0.60	278.83	279.15	279.46	279.76	280.05	280.33	280.60	280.86	281.12	281.37	281.61
0.80	278.83	279.22	279.59	279.96	280.30	280.64	280.97	281.28	281.59	281.88	282.17
1.00	278.83	279.29	279.73	280.15	280.56	280.95	281.33	281.70	282.05	282.39	282.72
1.20	278.83	279.36	279.86	280.35	280.81	281.26	281.69	282.11	282.50	282.89	283.26
1.40	278.83	279.43	280.00	280.54	281.07	281.56	282.05	282.51	282.95	283.37	283.78
1.60	278.83	279.50	280.13	280.74	281.31	281.87	282.40	282.90	283.39	283.85	284.30
1.80	278.83	279.57	280.26	280.93	281.56	282.16	282.74	283.29	283.82	284.32	284.80
2.00	278.83	279.63	280.39	281.12	281.81	282.46	283.08	283.68	284.24	284.78	285.30
2.20	278.83	279.70	280.52	281.31	282.05	282.75	283.42	284.05	284.65	285.23	285.78
2.40	278.83	279.77	280.66	281.49	282.29	283.04	283.75	284.42	285.06	285.67	286.25
2.60	278.83	279.84	280.79	281.68	282.53	283.32	284.08	284.79	285.46	286.10	286.71
2.80	278.83	279.91	280.91	281.87	282.76	283.60	284.40	285.15	285.85	286.53	287.16
3.00	278.83	279.98	281.04	282.05	283.00	283.88	284.72	285.50	286.24	286.94	287.60
3.20	278.83	280.04	281.17	282.23	283.23	284.15	285.03	285.85	286.62	287.35	288.03
3.40	278.83	280.11	281.30	282.41	283.46	284.42	285.34	286.19	286.99	287.74	288.45

Table F.3.b: Exit Temperature for a Shock-plug in K for values of Phi from -1.00 to 3.40, and Area Ratio from 0.9192 to 0.8621

Area Ratio/Phi	0.9192	0.9124	0.9058	0.8993	0.8929	0.8866	0.8803	0.8742	0.8681	0.8621
-1.00	276.55	276.38	276.23	276.08	275.93	275.79	275.66	275.54	275.42	275.31
-0.80	277.26	277.15	277.05	276.95	276.86	276.77	276.69	276.61	276.53	276.46
-0.60	277.95	277.90	277.85	277.80	277.76	277.72	277.68	277.65	277.62	277.59
-0.40	278.64	278.63	278.63	278.64	278.64	278.64	278.65	278.66	278.67	278.69
-0.20	279.30	279.35	279.40	279.45	279.50	279.55	279.60	279.65	279.70	279.75
0.00	279.96	280.06	280.15	280.24	280.34	280.43	280.52	280.61	280.70	280.79
0.20	280.60	280.74	280.88	281.02	281.15	281.29	281.42	281.54	281.67	281.79
0.40	281.23	281.42	281.60	281.78	281.95	282.12	282.29	282.45	282.61	282.76
0.60	281.85	282.07	282.30	282.51	282.72	282.93	283.13	283.32	283.52	283.70
0.80	282.45	282.72	282.98	283.23	283.48	283.72	283.95	284.17	284.39	284.61
1.00	283.04	283.34	283.64	283.93	284.21	284.48	284.74	285.00	285.24	285.48
1.20	283.61	283.96	284.29	284.61	284.92	285.22	285.51	285.79	286.06	286.33
1.40	284.18	284.56	284.92	285.27	285.61	285.94	286.25	286.56	286.85	287.14
1.60	284.73	285.14	285.53	285.91	286.28	286.63	286.97	287.30	287.61	287.92
1.80	285.26	285.71	286.13	286.54	286.93	287.30	287.66	288.01	288.35	288.67
2.00	285.79	286.26	286.71	287.14	287.56	287.95	288.33	288.70	289.05	289.38
2.20	286.30	286.80	287.27	287.73	288.16	288.58	288.97	289.36	289.72	290.07
2.40	286.80	287.32	287.82	288.30	288.75	289.18	289.59	289.99	290.36	290.72
2.60	287.28	287.83	288.35	288.85	289.32	289.76	290.19	290.59	290.98	291.34
2.80	287.76	288.33	288.87	289.38	289.86	290.32	290.76	291.17	291.57	291.94
3.00	288.22	288.81	289.37	289.89	290.39	290.86	291.30	291.73	292.12	292.50
3.20	288.67	289.28	289.85	290.39	290.90	291.38	291.83	292.25	292.66	293.03
3.40	289.11	289.73	290.32	290.87	291.38	291.87	292.32	292.75	293.16	293.54

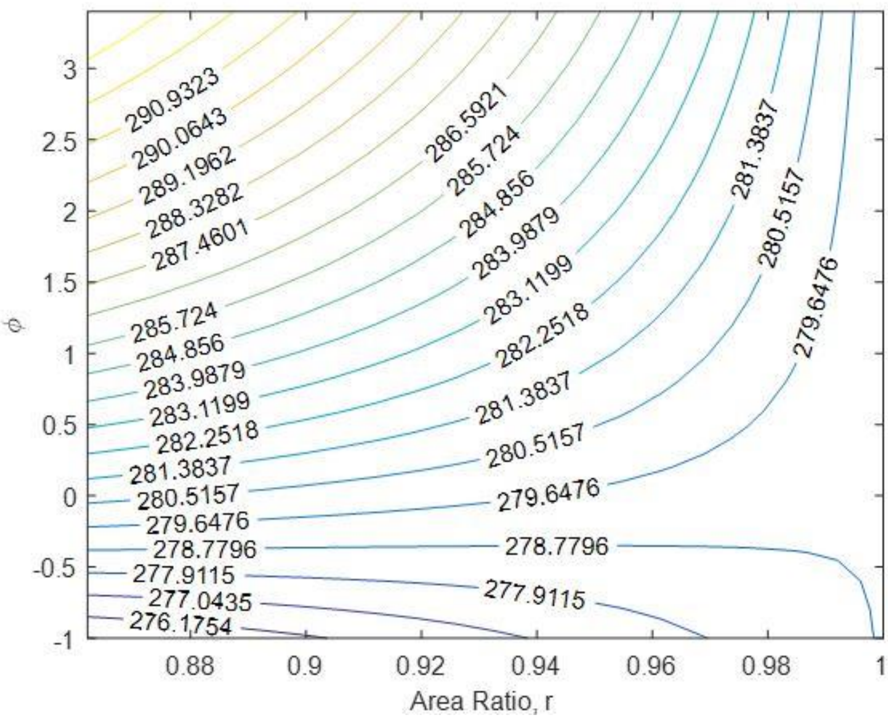

Figure F.3: Contour Plot of the Exit Temperature in K with Respect to Phi and Area Ratio

Table F.4.a: Back Pressure for a Shock-plug in Pa for values of Phi from -1.00 to 3.40, and Area Ratio from 1.000 to 0.9259

Area Ratio/Phi	1.0000	0.9921	0.9843	0.9766	0.9690	0.9616	0.9542	0.9470	0.9399	0.9329	0.9259
-1.00	75000	74085	73207	72352	71525	70730	69954	69202	68478	67770	67083
-0.80	75000	74175	73379	72603	71848	71119	70406	69712	69041	68384	67744
-0.60	75000	74265	73554	72856	72176	71517	70870	70238	69625	69023	68435
-0.40	75000	74356	73730	73114	72511	71924	71346	70781	70230	69687	69155
-0.20	75000	74447	73907	73374	72851	72340	71836	71340	70855	70377	69906
0.00	75000	74538	74086	73639	73198	72766	72338	71916	71503	71093	70690
0.20	75000	74630	74267	73906	73550	73201	72853	72510	72173	71838	71507
0.40	75000	74722	74449	74178	73909	73645	73382	73122	72866	72611	72359
0.60	75000	74815	74633	74453	74275	74100	73925	73753	73583	73414	73247
0.80	75000	74908	74819	74732	74647	74564	74483	74403	74325	74248	74173
1.00	75000	75001	75006	75014	75025	75039	75054	75072	75092	75114	75138
1.20	75000	75095	75196	75301	75411	75524	75641	75762	75886	76013	76144
1.40	75000	75190	75386	75591	75803	76019	76243	76473	76707	76947	77192
1.60	75000	75285	75579	75885	76202	76526	76861	77206	77556	77917	78286
1.80	75000	75380	75773	76183	76608	77044	77495	77960	78435	78924	79426
2.00	75000	75476	75969	76486	77021	77573	78146	78738	79344	79971	80616
2.20	75000	75572	76167	76792	77442	78114	78814	79539	80284	81058	81857
2.40	75000	75668	76366	77102	77870	78666	79499	80365	81257	82188	83152
2.60	75000	75765	76568	77416	78306	79231	80203	81216	82265	83363	84504
2.80	75000	75863	76771	77735	78749	79808	80924	82093	83307	84584	85916
3.00	75000	75961	76976	78058	79201	80398	81665	82997	84387	85853	87390
3.20	75000	76059	77183	78385	79660	81001	82426	83930	85505	87174	88932
3.40	75000	76158	77391	78716	80127	81617	83207	84891	86663	88549	90543

Table F.4.b: Back Pressure for a Shock-plug in Pa for values of Phi from -1.00 to 3.40, and Area Ratio from 0.9124 to 0.8621

Area Ratio/Phi	0.9192	0.9124	0.9058	0.8993	0.8929	0.8866	0.8803	0.8742	0.8681	0.8621
-1.00	66420	65772	65141	64532	63935	63353	62790	62238	61700	61179
-0.80	67125	66517	65924	65349	64785	64234	63700	63174	62660	62161
-0.60	67863	67301	66751	66216	65690	65175	64674	64180	63696	63225
-0.40	68636	68125	67624	67135	66652	66179	65717	65260	64812	64375
-0.20	69446	68991	68544	68107	67674	67249	66832	66420	66014	65617
0.00	70294	69902	69515	69136	68760	68389	68025	67664	67308	66959
0.20	71182	70858	70539	70225	69912	69604	69300	68998	68700	68406
0.40	72110	71863	71618	71376	71136	70897	70663	70429	70197	69969
0.60	73082	72918	72755	72594	72434	72276	72119	71963	71808	71655
0.80	74099	74026	73954	73883	73813	73744	73676	73608	73541	73476
1.00	75163	75189	75217	75246	75277	75308	75340	75374	75408	75443
1.20	76276	76411	76549	76689	76831	76976	77121	77269	77419	77570
1.40	77441	77695	77954	78216	78483	78754	79027	79306	79588	79872
1.60	78660	79045	79436	79833	80239	80652	81069	81496	81929	82366
1.80	79937	80463	81000	81546	82107	82679	83259	83854	84460	85074
2.00	81274	81954	82652	83363	84095	84846	85609	86396	87200	88018
2.20	82675	83523	84397	85290	86214	87164	88136	89140	90172	91226
2.40	84143	85175	86241	87336	88474	89649	90855	92107	93401	94728
2.60	85682	86914	88192	89510	90886	92314	93786	95323	96918	98563
2.80	87297	88746	90257	91823	93465	95177	96952	98814	100757	102773
3.00	88991	90678	92446	94286	96226	98259	100378	102614	104961	107412
3.20	90771	92718	94767	96913	99186	101582	104094	106761	109579	112540
3.40	92640	94871	97232	99717	102365	105172	108134	111300	114668	118234

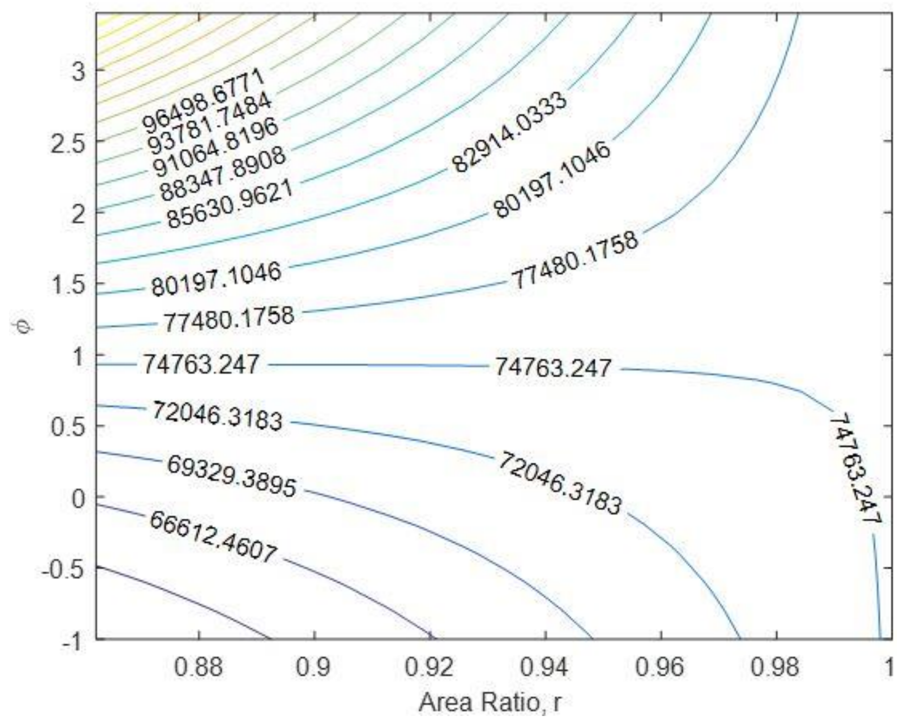

Figure F.4: Contour Plot of the Back Pressure in Pa with Respect to Phi and Area Ratio

Table F.5.a: Entropy Generation for a Shock-plug in kJ/kg-K for values of Phi from -1.00 to 3.40, and Area Ratio from 1.000 to 0.9259

Area Ratio/Phi	1.0000	0.9921	0.9843	0.9766	0.9690	0.9616	0.9542	0.9470	0.9399	0.9329	0.9259
-1.00	84.649	88.129	91.512	94.842	98.100	101.270	104.393	107.453	110.434	113.373	116.255
-0.80	84.649	87.786	90.844	93.862	96.824	99.715	102.569	105.374	108.112	110.819	113.480
-0.60	84.649	87.442	90.171	92.873	95.532	98.132	100.708	103.244	105.727	108.187	110.611
-0.40	84.649	87.096	89.494	91.874	94.221	96.524	98.809	101.065	103.279	105.477	107.647
-0.20	84.649	86.749	88.812	90.864	92.894	94.888	96.873	98.836	100.766	102.687	104.587
0.00	84.649	86.401	88.126	89.845	91.548	93.226	94.898	96.556	98.188	99.816	101.430
0.20	84.649	86.052	87.435	88.815	90.185	91.536	92.885	94.224	95.545	96.864	98.173
0.40	84.649	85.701	86.739	87.775	88.804	89.819	90.833	91.841	92.835	93.829	94.816
0.60	84.649	85.349	86.038	86.725	87.405	88.075	88.743	89.405	90.058	90.710	91.358
0.80	84.649	84.997	85.333	85.664	85.988	86.302	86.612	86.917	87.213	87.507	87.796
1.00	84.649	84.643	84.624	84.593	84.552	84.502	84.443	84.375	84.300	84.217	84.128
1.20	84.649	84.287	83.909	83.512	83.099	82.674	82.233	81.779	81.316	80.840	80.354
1.40	84.649	83.931	83.190	82.420	81.627	80.817	79.982	79.128	78.262	77.374	76.471
1.60	84.649	83.573	82.467	81.318	80.137	78.932	77.691	76.422	75.136	73.818	72.476
1.80	84.649	83.215	81.738	80.206	78.628	77.018	75.358	73.661	71.937	70.170	68.369
2.00	84.649	82.855	81.005	79.082	77.100	75.075	72.984	70.842	68.665	66.428	64.145
2.20	84.649	82.494	80.267	77.948	75.554	73.102	70.567	67.966	65.317	62.591	59.804
2.40	84.649	82.131	79.525	76.804	73.989	71.101	68.108	65.032	61.893	58.656	55.342
2.60	84.649	81.768	78.777	75.649	72.404	69.069	65.606	62.038	58.391	54.623	50.756
2.80	84.649	81.403	78.025	74.483	70.801	67.007	63.060	58.985	54.811	50.488	46.043
3.00	84.649	81.037	77.269	73.306	69.178	64.915	60.470	55.871	51.149	46.250	41.201
3.20	84.649	80.670	76.507	72.118	67.535	62.792	57.835	52.695	47.406	41.906	36.225
3.40	84.649	80.301	75.740	70.920	65.873	60.638	55.155	49.456	43.579	37.454	31.112

Table F.5.b: Entropy Generation for a Shock-plug in kJ/kg-K for values of Phi from -1.00 to 3.40, and Area Ratio from 0.9192 to 0.8621

Area Ratio/Phi	0.9192	0.9124	0.9058	0.8993	0.8929	0.8866	0.8803	0.8742	0.8681	0.8621
-1.00	119.066	121.840	124.564	127.222	129.848	132.428	134.949	137.441	139.892	142.289
-0.80	116.081	118.655	121.186	123.662	126.113	128.527	130.889	133.229	135.534	137.792
-0.60	112.986	115.340	117.661	119.935	122.192	124.418	126.600	128.767	130.904	133.002
-0.40	109.778	111.895	113.986	116.039	118.080	120.097	122.078	124.048	125.995	127.909
-0.20	106.456	108.317	110.159	111.970	113.774	115.560	117.317	119.067	120.800	122.505
0.00	103.019	104.605	106.176	107.725	109.270	110.801	112.311	113.817	115.310	116.782
0.20	99.465	100.755	102.036	103.300	104.562	105.816	107.053	108.289	109.517	110.728
0.40	95.791	96.766	97.734	98.691	99.647	100.598	101.538	102.477	103.411	104.334
0.60	91.996	92.634	93.268	93.894	94.520	95.142	95.756	96.371	96.982	97.586
0.80	88.078	88.358	88.634	88.905	89.174	89.440	89.701	89.961	90.219	90.473
1.00	84.034	83.933	83.828	83.718	83.604	83.485	83.363	83.238	83.109	82.978
1.20	79.861	79.357	78.845	78.329	77.802	77.269	76.733	76.189	75.639	75.087
1.40	75.558	74.627	73.682	72.731	71.763	70.784	69.800	68.801	67.792	66.781
1.60	71.121	69.737	68.333	66.919	65.477	64.019	62.552	61.061	59.554	58.040
1.80	66.547	64.685	62.793	60.885	58.938	56.964	54.977	52.952	50.903	48.843
2.00	61.834	59.466	57.057	54.622	52.134	49.608	47.059	44.459	41.821	39.164
2.20	56.976	54.075	51.117	48.123	45.057	41.938	38.785	35.560	32.283	28.975
2.40	51.972	48.507	44.969	41.379	37.696	33.940	30.135	26.236	22.264	18.244
2.60	46.816	42.758	38.603	34.380	30.037	25.598	21.091	16.461	11.733	6.935
2.80	41.505	36.820	32.014	27.116	22.068	16.896	11.631	6.209	0.657	-4.992
3.00	36.035	30.688	25.191	19.576	13.774	7.814	1.732	-4.550	-11.001	-17.585
3.20	30.399	24.355	18.126	11.747	5.138	-1.668	-8.634	-15.852	-23.285	-30.897
3.40	24.593	17.814	10.809	3.617	-3.856	-11.574	-19.498	-27.733	-36.245	-44.991

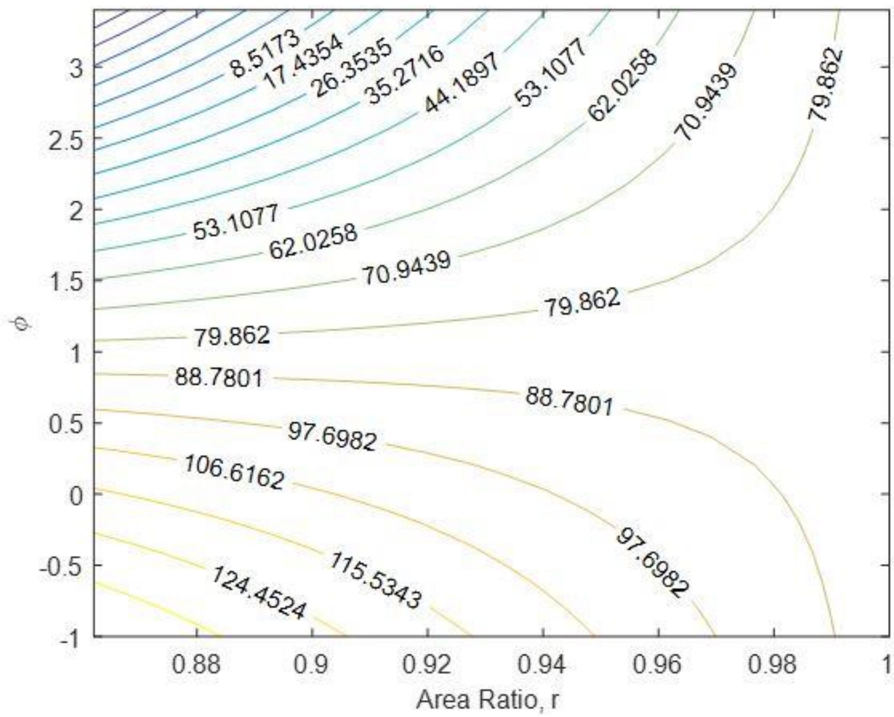

Figure F.5: Contour Plot of the Entropy Generation in kJ/kg-K with Respect to Phi and Area Ratio

Table F.6.a: Side Pressure for a Shock-plug in Pa for values of Phi from -1.00 to 3.40, and Area Ratio from 1.000 to 0.9259

Area Ratio/Phi	1.0000	0.9921	0.9843	0.9766	0.9690	0.9616	0.9542	0.9470	0.9399	0.9329	0.9259
-1.00	-31747	-30845	-29983	-29148	-28345	-27576	-26830	-26111	-25423	-24754	-24109
-0.80	-22596	-21973	-21376	-20795	-20234	-19694	-19170	-18663	-18175	-17699	-17239
-0.60	-13446	-13053	-12674	-12304	-11946	-11600	-11263	-10935	-10618	-10309	-10008
-0.40	-4295	-4083	-3877	-3675	-3479	-3289	-3102	-2920	-2744	-2570	-2401
-0.20	4856	4937	5016	5094	5171	5245	5319	5391	5461	5530	5598
0.00	14006	14006	14006	14006	14006	14006	14006	14006	14006	14006	14006
0.20	23157	23125	23094	23062	23031	22999	22967	22936	22904	22873	22841
0.40	32307	32295	32280	32265	32248	32229	32210	32189	32168	32145	32121
0.60	41458	41514	41566	41615	41660	41702	41741	41777	41810	41840	41867
0.80	50609	50784	50953	51116	51273	51423	51569	51709	51844	51974	52100
1.00	59759	60106	60440	60768	61088	61397	61701	61997	62284	62567	62842
1.20	68910	69478	70030	70575	71110	71631	72147	72653	73147	73636	74117
1.40	78060	78901	79724	80539	81343	82131	82914	83687	84447	85204	85952
1.60	87211	88376	89521	90661	91790	92902	94012	95114	96203	97290	98372
1.80	96362	97903	99423	100943	102455	103952	105452	106947	108430	109920	111408
2.00	105512	107481	109432	111389	113344	115286	117241	119199	121149	123116	125091
2.20	114663	117112	119547	122000	124459	126912	129391	131884	134378	136906	139454
2.40	123814	126795	129770	132778	135806	138837	141913	145019	148140	151316	154532
2.60	132964	136531	140103	143727	147388	151068	154818	158619	162455	166376	170365
2.80	142115	146320	150545	154848	159210	163613	168116	172701	177347	182118	186993
3.00	151265	156162	161099	166144	171278	176479	181821	187283	192841	198574	204460
3.20	160416	166058	171764	177617	183595	189675	195945	202383	208963	215781	222815
3.40	169567	176007	182543	189270	196167	203210	210501	218020	225741	233777	242110

Table F.6.b: Side Pressure for a Shock-plug in Pa for values of Phi from -1.00 to 3.40, and Area Ratio from 0.9192 to 0.8621

Area Ratio/Phi	0.9192	0.9124	0.9058	0.8993	0.8929	0.8866	0.8803	0.8742	0.8681	0.8621
-1.00	-23489	-22888	-22306	-21747	-21204	-20678	-20172	-19679	-19202	-18742
-0.80	-16796	-16364	-15945	-15541	-15147	-14764	-14395	-14034	-13684	-13345
-0.60	-9718	-9433	-9156	-8889	-8626	-8371	-8124	-7881	-7645	-7416
-0.40	-2237	-2076	-1918	-1765	-1615	-1468	-1325	-1184	-1046	-912
-0.20	5664	5730	5794	5857	5918	5979	6039	6097	6155	6211
0.00	14006	14006	14006	14006	14006	14006	14006	14006	14006	14006
0.20	22810	22778	22747	22715	22684	22653	22622	22591	22560	22529
0.40	32097	32072	32045	32019	31991	31962	31933	31904	31874	31843
0.60	41892	41915	41935	41953	41969	41982	41994	42004	42012	42018
0.80	52221	52338	52450	52559	52664	52765	52862	52956	53047	53134
1.00	63110	63373	63630	63880	64126	64367	64602	64832	65059	65279
1.20	74588	75055	75515	75965	76412	76852	77285	77714	78138	78554
1.40	86688	87422	88149	88866	89581	90291	90991	91691	92386	93072
1.60	99443	100515	101582	102640	103701	104760	105810	106865	107918	108965
1.80	112888	114377	115867	117351	118846	120346	121841	123351	124866	126380
2.00	127063	129056	131061	133067	135099	137146	139197	141279	143380	145489
2.20	142009	144605	147228	149865	152549	155268	158005	160798	163632	166492
2.40	157773	161080	164438	167830	171300	174833	178410	182078	185820	189618
2.60	174402	178542	182767	187055	191466	195979	200574	205312	210175	215139
2.80	191950	197059	202298	207645	213173	218861	224685	230727	236966	243375
3.00	210475	216705	223127	229715	236566	243656	250960	258584	266508	274704
3.20	230039	237560	245354	253396	261807	270566	279646	289189	299175	309581
3.40	250711	259713	269096	278834	289081	299822	311033	322900	335412	348553

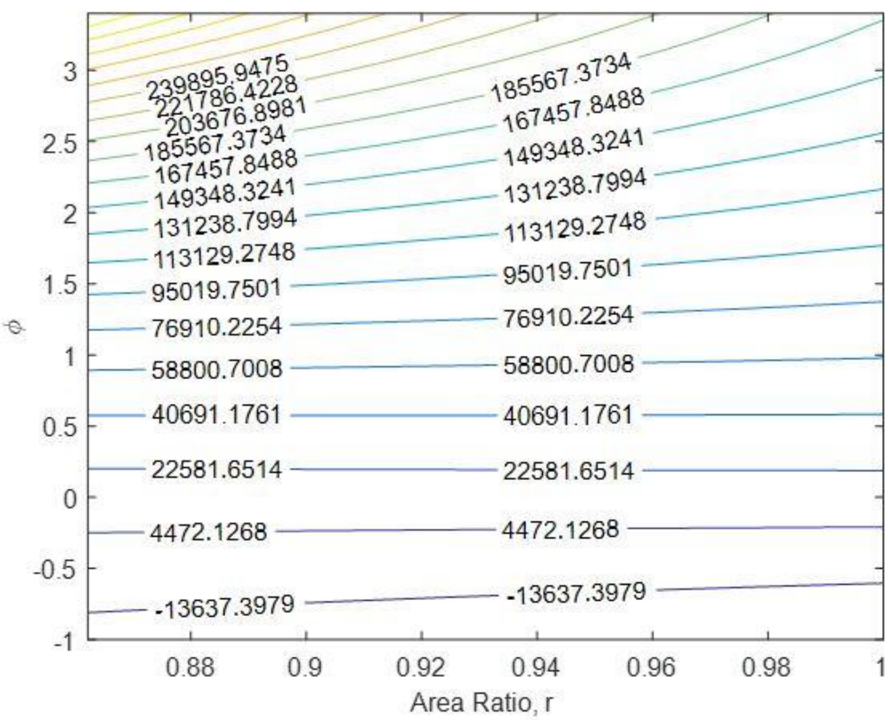

Figure F.6: Contour Plot of the Side Pressure in Pa with Respect to Phi and Area Ratio

Appendix G

This appendix shows the main code and data that is used to create Figures 4.11 through 4.17. ShockThickness.m is the main function that creates the data although it also calls ShockwaveSolver.m, FixedThicknessNoPhiShockPlugSolver.m, and FixedThicknessShockPlugSolver.m as shown in Appendix D.

```
%ShockThickness.m
%
%This program finds the location and flow propteries of a shockwave and
%a shock-plug. It uses the information from the NozzleGeometry
%function, was well as several flow properties. This program finds the
%location of both shocks by solving for a fixed back pressure.
%
% -----------------------Function Passthrough Inputs-----------------------
%
%To is the total temperature of the flow when on the leading edge of the
%shockwave in Kelvin
%
%Po is the total pressure of the flow on the leading edge of the shockave
%in Pa.
%
%Gamma is the raio of specific heats. This is normally 1.4, but is left as
%an input for the sake of robustness.
%
%R is the specific gas constant of the working fluid in the flow. This is
%in units of J/kg-K
%
```

```
%Cp is the specific heat with constant pressure of the working fluid. This
%is inJ/kg-k
%
%Index is the number of cells in the nozzle geometry outputs. A higher
%number leads to a higher resolution of the nozzle.
%
%Muo is the reference mu value for the Sutherland Equation to calculate the
%viscosity of the flow at any point.
%
%Tref is the reference temperature value for the Sutherland Equation to
%calculate the viscosity of the flow at any point.
%
%S is another reference value for the Sutherland Equation to calculate the
%viscosity of the flow at any point.
%
%ShockLocWave/Plug is an interger number that indexes where the shock is
%located in the LengthVector. The -Wave and -Plug suffixes differentiate
%between the type of shock.
%
%Phi is a unitless parameter from zero to one that solves for the side
%pressure of the shockplug. In practice, these bounds do not hold.
%
% ----------------------Function Passthrough Outputs-----------------------
%
%Astar is the area of the throat of the nozzle in units of m^2.
%
%AreaVector is a vector of the areas as you progress down the nozzle. The
%first index in this vector should be the throat area, while the last index
%is the area at the exit of the nozzle. All entries are in m^2
```

```
%
%LengthVector is the vector that contains the position from the throat.
%This matches up to the AreaVector. The first entry is zero as it is the
%throat.
%
%MxWave/Plug is the Mach number just before the shockwave. The -Wave and
%-Plug suffixes differentiate between the type of shock.
%
%PxWave/Plug is the pressure just before the shockwave in Pa.The -Wave and
%-Plug suffixes differentiate between the type of shock.
%
%TxWave/Plug is the temperture just before the shockwave in K.The -Wave and
%-Plug suffixes differentiate between the type of shock.
%
%MyWave/Plug is the Mach number just after the shockwaveThe -Wave and
%-Plug suffixes differentiate between the type of shock.
%
%PyWave/Plug is the pressure just after the shockwave in Pa.The -Wave and
%-Plug suffixes differentiate between the type of shock.
%
%TyWave/Plug is the temperture just after the shockwave in K.The -Wave and
%-Plug suffixes differentiate between the type of shock.
%
%PbWave/Plug is the pressure at the exit of the nozzle in Pa at the guessed
%shock location.The -Wave and -Plug suffixes differentiate between the type
%of shock.
%
%SidePressure is the pressure that is present that is normal to the
%nozzle's boundry inside of the shockplug.
```

```
%
%r is the area ratio of the nozzle section that is being analyized. defined
%as the area of the leading edge of the shockplug over the trailing edge of
%the shock plug.

%% Initialization Code

clc
clear variables
close all

%% -----------------------Constants and Variables--------------------------

%These are gas and flow properties used throughout the code.
Po = 101.3*10^3; %Pa
To = 298; %K
Gamma = 1.4;
R = 286.9; %J/kg K Gas Constant of Air
Cp = 1005.; %J/kg K Specific Heat of Air at 20C
d = 3.711*10^-10; %m diameter of air molecules
kB = 1.3806488*10^-23; %JK Boltzmann constant
Pb = 75*10^3; %Set back pressure in Pa

%This is a set of constants that are used to shorten the following
%equations. These come from reacuring parts of the shock relations.
A = sqrt(Gamma/(To*R))*Po;
B = 0.5*(Gamma-1);
C = -(Gamma+1)/(2*Gamma-2);
```

```
%% ---------------------------Nozzle Geometry--------------------------------

%This section calls the NozzleGeomtry function and produces legnth, area
%and width vectors of the nozzle.
Index = 100000;
[AreaVector,LengthVector,Width,AStar,dx] = NozzleGeometry(Index);

%% -------------------------ShockWave Solution------------------------------

%This section uses an inital guess of the location of the shockwave to
%produce a back prressure. This back pressure is compared to the desired
%back pressure, set in the Constants and Variables section. A new location
%is calcualted then fed back into the function to get a new back pressure.
%This contuinues until the difference between the calculated and guessed
%back pressure is within .1%.
Error = Pb;
i=1;
ShockLocWave = 8000;
DShockLocWave = 1;

while abs(Error) > 10
    i



[MxWave,PxWave,TxWave,MyWave,PyWave,TyWave,PbWave,SgenWave]=ShockwaveSolver(To
,Po,Gamma,R,AStar,AreaVector,ShockLocWave);
```

```
[DMxWave,DPxWave,DTxWave,DMyWave,DPyWave,DTyWave,DPbWave,DSgenWave]=ShockwaveS
olver(To,Po,Gamma,R,AStar,AreaVector,ShockLocWave+DShockLocWave);


    dPdShockLoc = (DPbWave-PbWave)/DShockLocWave;
    Error = (Pb-PbWave)
    ShockLocWave = ShockLocWave + int16(Error/dPdShockLoc);
    i = i+1;
end

%% ----------Calculation of S-P Properties with no Side Pressure-----------

%This section of the code solves for the maximum entropy generation at each
%value fo the shock-plug thickness. This is done by calling the
%FixedThicknessNoPhiShockPlugPover.m function. The shock wave solution is
%the leading or trailing edge of this shock-plug depending on the called
%function.

%Creates an array that represents the thickness of the shock-plug
ThicknessLength = 41;
MaxThickness = 1*10^-7;
%MaxThickness = 5*10^-9;
Thickness = linspace(0,MaxThickness,ThicknessLength);

%Intialization of the data that will be calculated later.
MxNoPS = zeros(1,length(Thickness));
PxNoPS = zeros(1,length(Thickness));
```

```
TxNoPS = zeros(1,length(Thickness));
MyNoPS = zeros(1,length(Thickness));
PyNoPS = zeros(1,length(Thickness));
TyNoPS = zeros(1,length(Thickness));
PbNoPS = zeros(1,length(Thickness));
rNoPS = zeros(1,length(Thickness));
SgenNoPS = zeros(1,length(Thickness));

%This look calls the function for each thickness value and output the
%proper flow parameters
for j = 1:ThicknessLength

    j



[MxNoPS(j),PxNoPS(j),TxNoPS(j),MyNoPS(j),PyNoPS(j),TyNoPS(j),PbNoPS(j),rNoPS(j
),SgenNoPS(j)] =
FixedThicknessNoPhiShockPlugSolver(To,Po,Gamma,R,Cp,AStar,AreaVector,LengthVec
tor,ShockLocWave,Thickness(j));
end



%Plots of the back pressure and entropy generation compared to the shock
%wave solution.
figure
plot(rNoPS,PbNoPS)
hold on
plot(rNoPS,ones(1,ThicknessLength)*PbWave,':')
```

```
hold off
%title('backpressue versus thickness for Maximum')
xlabel('Area Ratio')
ylabel('Back pressure (Pa)')
legend('Shock-plug Back Pressure','Shock Wave Back Pressure')

figure
plot(rNoPS,SgenNoPS)
hold on
plot(rNoPS,ones(1,ThicknessLength)*SgenWave,:)
xlabel('Area Ratio')
ylabel('Entropy Generation (kJ/kg-K)')
%title('r Vs Entropy Generation')
legend('Max Shock-plug Entropy Generation','Shock Wave Entropy Generation')



%% -------------Calculation of S-P with Sw Trailing edge-------------------

%This section of the code will solve for the value of Phi that matches the
%back pressure create by the shock-plug to the shock wave. This is done by
%calling the FixedThicknessShockPlugSolver.m function multiple times and
%using a simple root finding technique.

%Initialization of the values that will be calculated with the called
%function.
MxLE = zeros(1,length(Thickness));
PxLE = zeros(1,length(Thickness));
TxLE = zeros(1,length(Thickness));
```

```
MyLE = zeros(1,length(Thickness));
PyLE = zeros(1,length(Thickness));
TyLE = zeros(1,length(Thickness));
PbLE = zeros(1,length(Thickness));
SidePressureLE = zeros(1,length(Thickness));
rLE = zeros(1,length(Thickness));
SgenLE = zeros(1,length(Thickness));
Phi = zeros(1,length(Thickness));

%This is a step in Phi to produce an estimate of hte first derivative of
%the output.
DPhi = 0.001;

%Initialization of the data since an area ratio of 1 leads to the shock
%wave solution.
MxLE(1) = MxWave;
PxLE(1) = PxWave;
TxLE(1) = TxWave;
MyLE(1) = MyWave;
PyLE(1) = PyWave;
TyLE(1) = TyWave;
PbLE(1) = PbWave;
rLE(1) = 1;
SgenLE(1) = SgenWave;

%This loops calculates the phi value that results in a matched back
%pressure over a range of thickness values.
for k = 2:length(Thickness)
```

```
    k

    Error = Pb;
    Phi(k)=Phi(k-1);
    while abs(Error) >= 0.01

[MxLE(k),PxLE(k),TxLE(k),MyLE(k),PyLE(k),TyLE(k),PbLE(k),SidePressureLE(k),rLE
(k),SgenLE(k)] =
FixedThicknessShockPlugSolver(To,Po,Gamma,R,AStar,AreaVector,LengthVector,Shoc
kLocWave,Phi(k),Thickness(k));

        %This creates the first derivative estimate.

[DMxLE,DPxLE,DTxLE,DMyLE,DPyLE,DTyLE,DPbLE,DSidePressureLE,DrLE,DSgenLE] =
FixedThicknessShockPlugSolver(To,Po,Gamma,R,AStar,AreaVector,LengthVector,Shoc
kLocWave,Phi(k)+DPhi,Thickness(k));

        %This section calucates the error of the guessed phi value and
        %provides an other guess.
        dPdPhi = (DPbLE-PbLE(k))/DPhi;
        Error = Pb-PbLE(k)

        Phi(k) = Error/dPdPhi+Phi(k);
    end
```

```
end

%This calcualtes the minimum Phi value that can be obtained by each
%shock-plug that was calculated in the loop above. This is mostly to
%provide a real world constraint on Phi.
MinPhi = PxLE./(PxLE-PyLE);

%Plots of the resulting flow parameters as the thickness increases.
figure
plot(rLE(2:length(rLE)),Phi(2:length(rLE)))
hold on
plot(rLE(2:length(rLE)),MinPhi(2:length(rLE)))
xlabel('Area Ratio')
ylabel('Phi')
%title('Phi Vs r')
legend('Phi','Minimum Phi')

figure
plot(rLE(2:length(rLE)),SidePressureLE(2:length(rLE)))
xlabel('Area Ratio')
ylabel('Side Pressure (Pa)')
%title('Side Pressure Vs r')
```

The following tables show the data that is used to create the Figures 4.11 Through 4.17 in Sections 4.6 and 4.7

Table G.1.a: Values of Flow Properties for a Shock-Plug with No Side Pressure and the Shock Wave Solution is at the Trailing Edge for an Area Ratio from 1.0000 to 0.9200

Area Ratio	1.0000	0.9920	0.9840	0.9760	0.9680	0.9600	0.9520	0.9440	0.9360	0.9280	0.9200
Mx	1.9494	1.9395	1.9295	1.9194	1.9091	1.8988	1.8883	1.8776	1.8668	1.8558	1.8447
Px (Pa)	14006	14222	14443	14671	14906	15146	15394	15650	15912	16184	16463
Tx (K)	169.32	170.06	170.81	171.58	172.36	173.15	173.95	174.77	175.60	176.46	177.32
My	0.5863	0.5882	0.5900	0.5919	0.5939	0.5959	0.5980	0.6001	0.6023	0.6045	0.6069
Py (Pa)	59757	59560	59361	59157	58948	58737	58520	58298	58073	57841	57604
Ty (K)	278.84	278.73	278.62	278.50	278.38	278.26	278.13	277.99	277.86	277.72	277.57
Back Pressure (Pa)	74999	74855	74710	74561	74410	74257	74101	73941	73780	73615	73446
Sgen (kJ/kg-K)	84.653	85.197	85.748	86.312	86.888	87.471	88.070	88.681	89.301	89.937	90.587

Table G.1.b: Values of Flow Properties for a Shock-Plug with No Side Pressure and the Shock Wave Solution is at the Trailing Edge for an Area Ratio from 0.9121 to 0.8401

Area Ratio	0.9121	0.9040	0.8960	0.8881	0.8801	0.8720	0.8641	0.8561	0.8481	0.8401
Mx	1.8334	1.8219	1.8102	1.7985	1.7864	1.7741	1.7617	1.7490	1.7361	1.7230
Px (Pa)	16750	17048	17356	17672	18001	18340	18690	19054	19432	19822
Tx (K)	178.20	179.10	180.02	180.95	181.90	182.88	183.87	184.88	185.92	186.98
My	0.6092	0.6117	0.6143	0.6169	0.6196	0.6224	0.6253	0.6284	0.6315	0.6348
Py (Pa)	57363	57115	56861	56603	56336	56062	55783	55495	55199	54896
Ty (K)	277.42	277.27	277.11	276.94	276.77	276.59	276.40	276.21	276.00	275.79
Back Pressure (Pa)	73275	73100	72921	72740	72554	72364	72171	71973	71770	71564
Sgen (kJ/kg-K)	91.247	91.926	92.620	93.325	94.051	94.794	95.551	96.330	97.130	97.944

Table G.2.a: Values of Flow Properties for a Shock-Plug with No Side Pressure and the Shock Wave Solution is at the Leading Edge for an Area Ratio from 1.0000 to 0.9259

Area Ratio	1.0000	0.9921	0.9843	0.9766	0.9690	0.9616	0.9542	0.9470	0.9399	0.9329	0.9259
Mx	1.9494	1.9494	1.9494	1.9494	1.9494	1.9494	1.9494	1.9494	1.9494	1.9494	1.9494
Px (Pa)	14006	14006	14006	14006	14006	14006	14006	14006	14006	14006	14006
Tx (K)	169.32	169.32	169.32	169.32	169.32	169.32	169.32	169.32	169.32	169.32	169.32
My	0.5863	0.5863	0.5863	0.5863	0.5863	0.5863	0.5863	0.5863	0.5863	0.5863	0.5863
Py (Pa)	59757	59282	58817	58357	57904	57460	57021	56588	56165	55745	55332
Ty (K)	169.32	169.32	169.32	169.32	169.32	169.32	169.32	169.32	169.32	169.32	169.32
Back Pressure (Pa)	74999	74396	73806	73222	72647	72084	71526	70976	70439	69905	69380
Sgen (kJ/kg-K)	84.653	86.942	89.200	91.454	93.690	95.895	98.097	100.282	102.438	104.590	106.727

Table G.2.b: Values of Flow Properties for a Shock-Plug with No Side Pressure and the Shock Wave Solution is at the Leading Edge for an Area Ratio from 0.9192 to 0.8621

Area Ratio	0.9192	0.9124	0.9058	0.8993	0.8929	0.8866	0.8803	0.8742	0.8681	0.8621
Mx	1.9494	1.9494	1.9494	1.9494	1.9494	1.9494	1.9494	1.9494	1.9494	1.9494
Px (Pa)	14006	14006	14006	14006	14006	14006	14006	14006	14006	14006
Tx (K)	169.32	169.32	169.32	169.32	169.32	169.32	169.32	169.32	169.32	169.32
My	0.5863	0.5863	0.5863	0.5863	0.5863	0.5863	0.5863	0.5863	0.5863	0.5863
Py (Pa)	54926	54525	54129	53742	53357	52978	52607	52238	51875	51519
Ty (K)	169.32	169.32	169.32	169.32	169.32	169.32	169.32	169.32	169.32	169.32
Back Pressure (Pa)	68865	68355	67852	67360	66871	66389	65917	65449	64987	64534
Sgen (kJ/kg-K)	108.834	110.940	113.029	115.092	117.151	119.197	121.215	123.232	125.234	127.211

Table G.3.a: Values of Flow Properties for a Shock-Plug with a matched Phi Value and the Shock Wave Solution at the Trailing Edge for an Area Ratio from 1.0000 to 0.9200

Area Ratio	1.0000	0.9920	0.9840	0.9760	0.9680	0.9600	0.9520	0.9440	0.9360	0.9280	0.9200
Phi	0.0000	-0.0018	-0.0046	-0.0073	-0.0101	-0.0129	-0.0160	-0.0192	-0.0225	-0.0260	-0.0297
Mx	1.949	1.939	1.930	1.919	1.909	1.899	1.888	1.878	1.867	1.856	1.845
Px (Pa)	14006	14222	14443	14671	14906	15146	15394	15650	15912	16184	16463
Tx (K)	169.32	170.06	170.81	171.58	172.36	173.15	173.95	174.77	175.60	176.46	177.32
My	0.5863	0.5863	0.5863	0.5863	0.5863	0.5863	0.5863	0.5863	0.5863	0.5863	0.5863
Py (Pa)	59759	59760	59760	59760	59760	59760	59760	59760	59760	59760	59760
Ty (K)	278.83	278.83	278.83	278.83	278.83	278.83	278.83	278.83	278.83	278.83	278.83
Back Pressure (Pa)	75000	75000	75000	75000	75000	75000	75000	75000	75000	75000	75000
Sgen (kJ/kg-K)	84.649	84.648	84.648	84.648	84.648	84.648	84.648	84.648	84.648	84.648	84.648
Side Pressure (Pa)	0	14140	14236	14343	14454	14568	14685	14805	14927	15051	15179

Table G.3.b: Values of Flow Properties for a Shock-Plug with a matched Phi Value and the Shock Wave Solution at the Trailing Edge for an Area Ratio from 0.9121 to 0.8401

Area Ratio	0.9121	0.9040	0.8960	0.8881	0.8801	0.8720	0.8641	0.8561	0.8481	0.8401
Phi	-0.0335	-0.0376	-0.0420	-0.0465	-0.0514	-0.0565	-0.0619	-0.0677	-0.0738	-0.0803
Mx	1.833	1.822	1.810	1.798	1.786	1.774	1.762	1.749	1.736	1.723
Px (Pa)	16750	17048	17356	17672	18001	18340	18690	19054	19432	19822
Tx (K)	178.20	179.10	180.02	180.95	181.90	182.88	183.87	184.88	185.92	186.98
My	0.5863	0.5863	0.5863	0.5863	0.5863	0.5863	0.5863	0.5863	0.5863	0.5863
Py (Pa)	59760	59760	59760	59760	59760	59760	59760	59760	59760	59760
Ty (K)	278.83	278.83	278.83	278.83	278.83	278.83	278.83	278.83	278.83	278.83
Back Pressure (Pa)	75000	75000	75000	75000	75000	75000	75000	75000	75000	75000
Sgen (kJ/kg-K)	84.648	84.648	84.648	84.648	84.648	84.648	84.648	84.648	84.648	84.648
Side Pressure (Pa)	15308	15441	15576	15714	15856	16001	16148	16300	16455	16613